A population dependent diffusion model (PDM)

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Introduction

- **Aim: the development of a diffusion model for demand estimation and forecasting, taking into account the size of population as a decision variable.**

- **Innovation:**
  - The introduction of the “population” diffusion model (PDM).

- **Evaluation over historical data from the wider European area.**

Diffusion Models

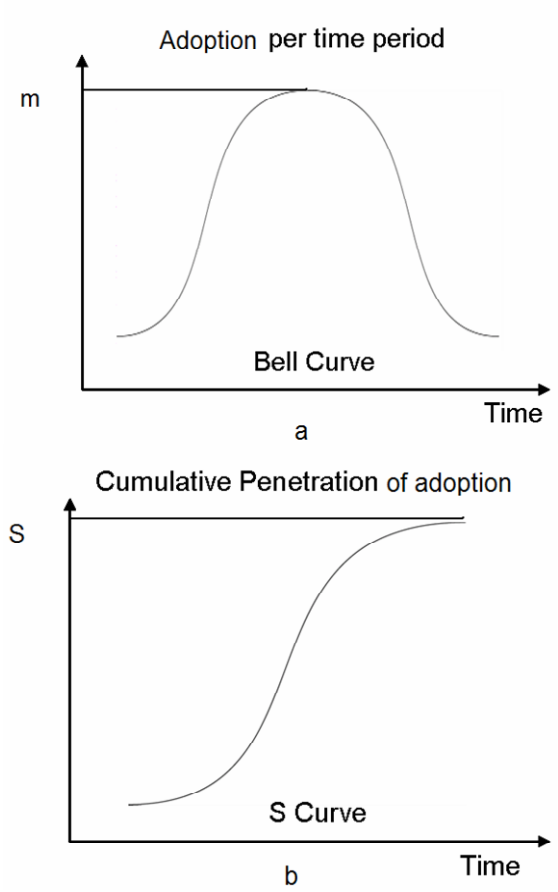
- **Mathematical functions of time**, used to estimate the diffusion process and a product's life cycle.
- **General formulation:**

$$\frac{dN(t)}{dt} = \delta * N(t) * [S - N(t)]$$

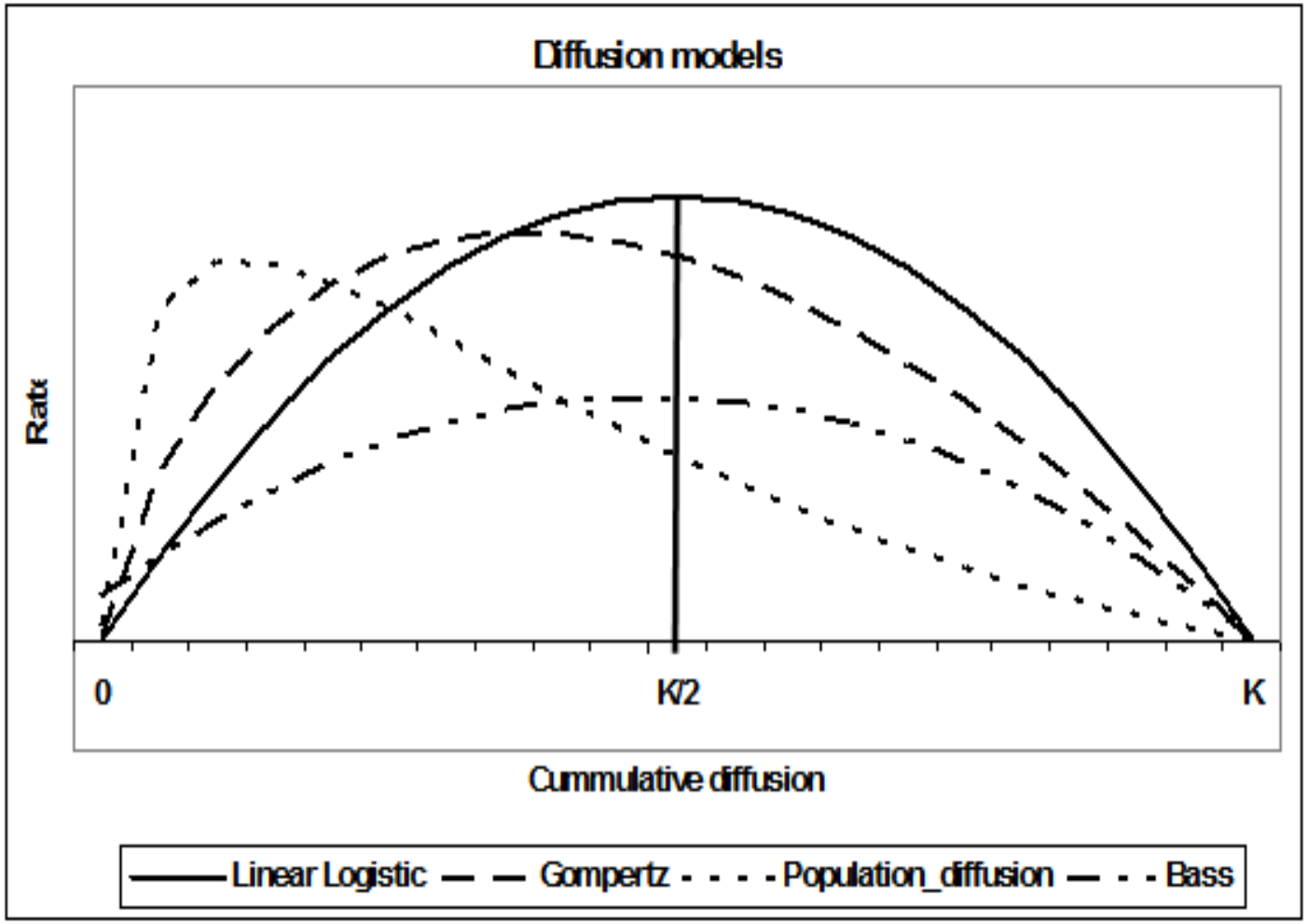
- **Commonly Used Models:**
  - Bass model
  - Fisher – Pry model
  - Logistic family models
  - Gompertz model

Idealized diffusion process

- a: Adoption per time period (Bell curve),
- b: Cumulative penetration of adoption (S-Curve)



The need for a new model



Model development

- **Logistic model**

$$\frac{dN(t)}{dt} = rN(t)\left(K - N(t)\right)$$

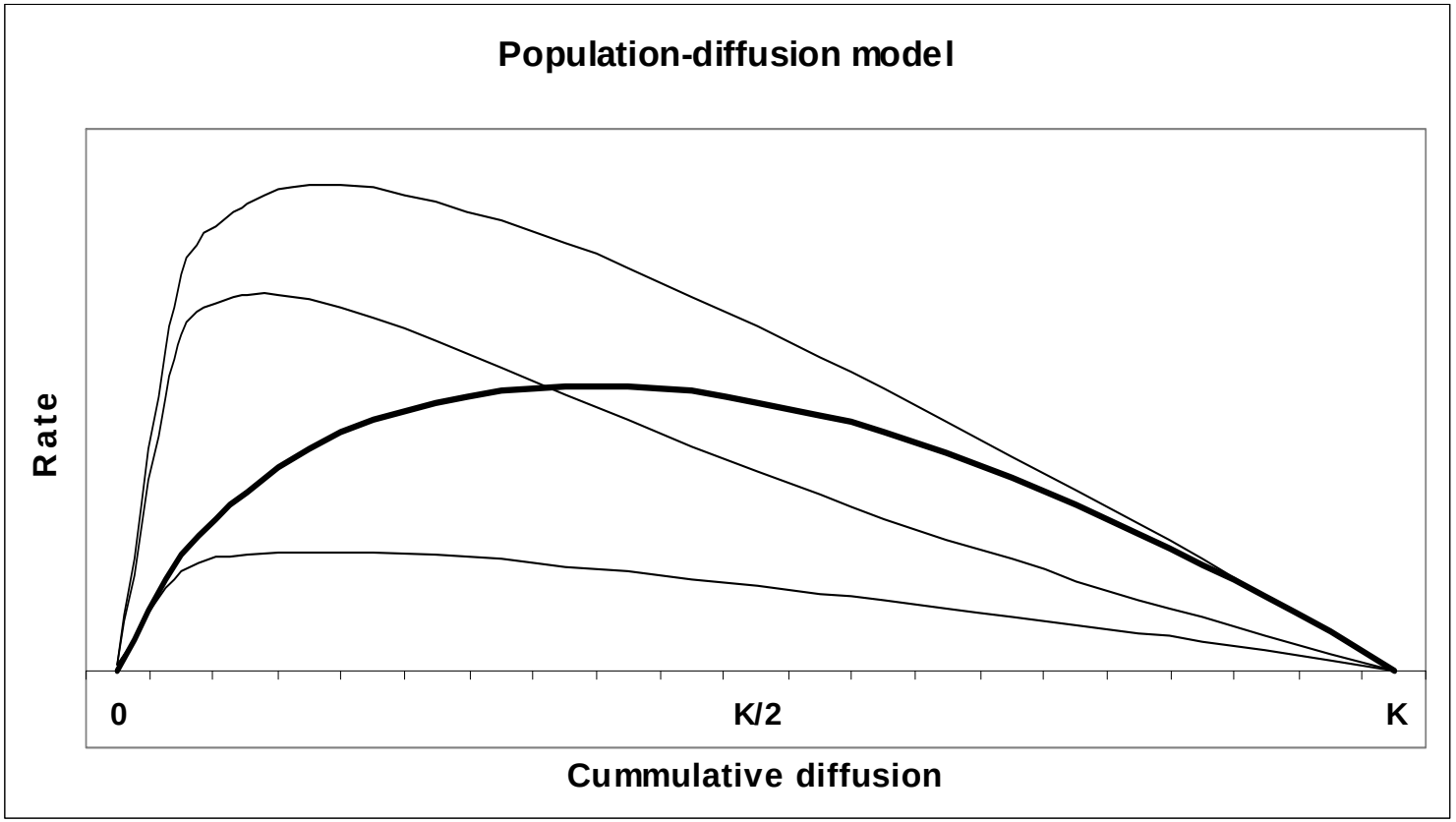
- **Gompertz model**

$$\frac{dN(t)}{dt} = rN(t)\ln\left(\frac{K}{N(t)}\right)$$

- **PDM**

$$\frac{dN(t)}{dt} = rN(t)\ln\left(a + b\frac{P}{N(t)}\right)\ln\left(\frac{K}{N(t)}\right)$$

Diffusion shape of the PDM for different values of parameters and the same K

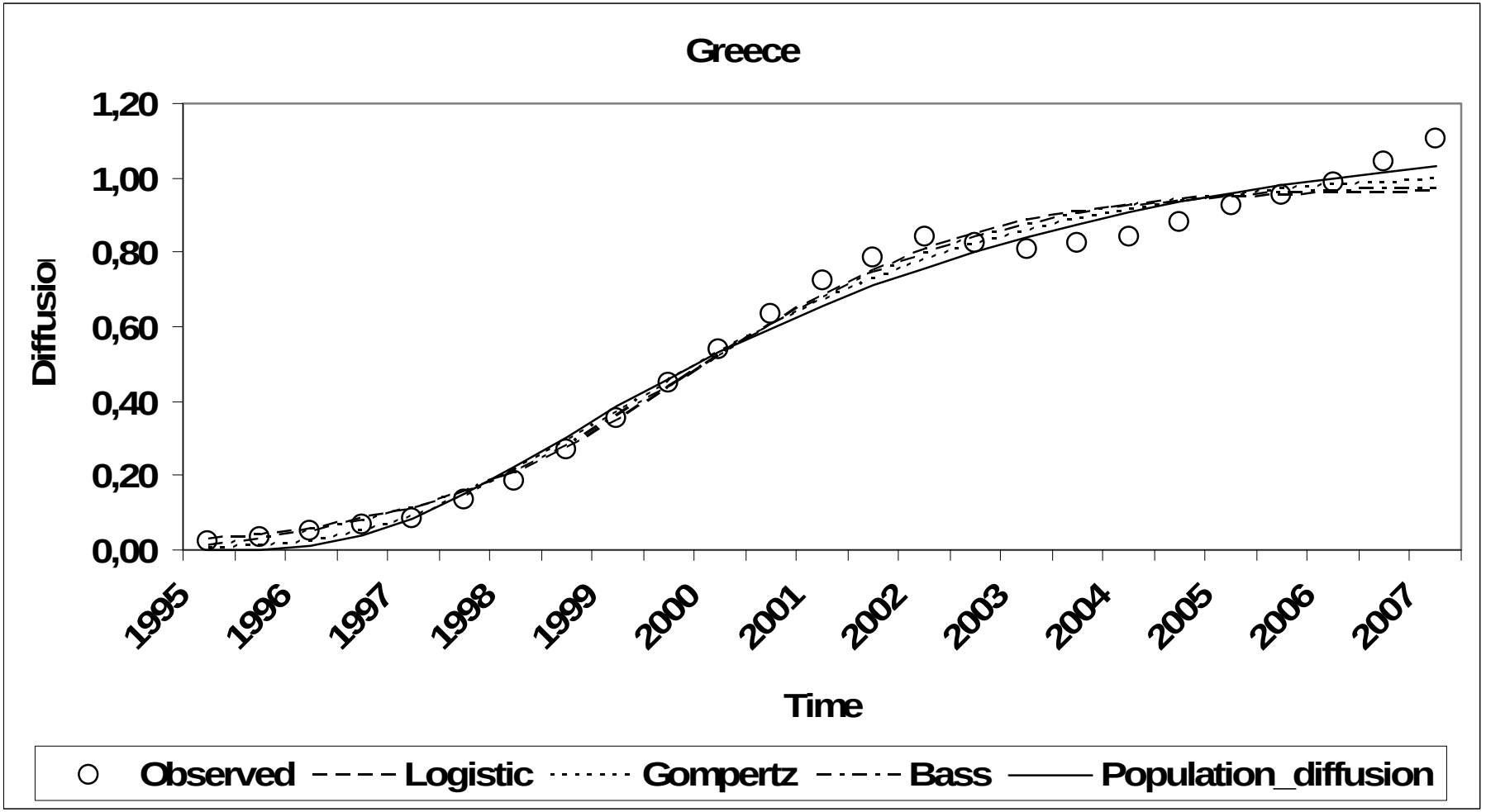


Model formulation

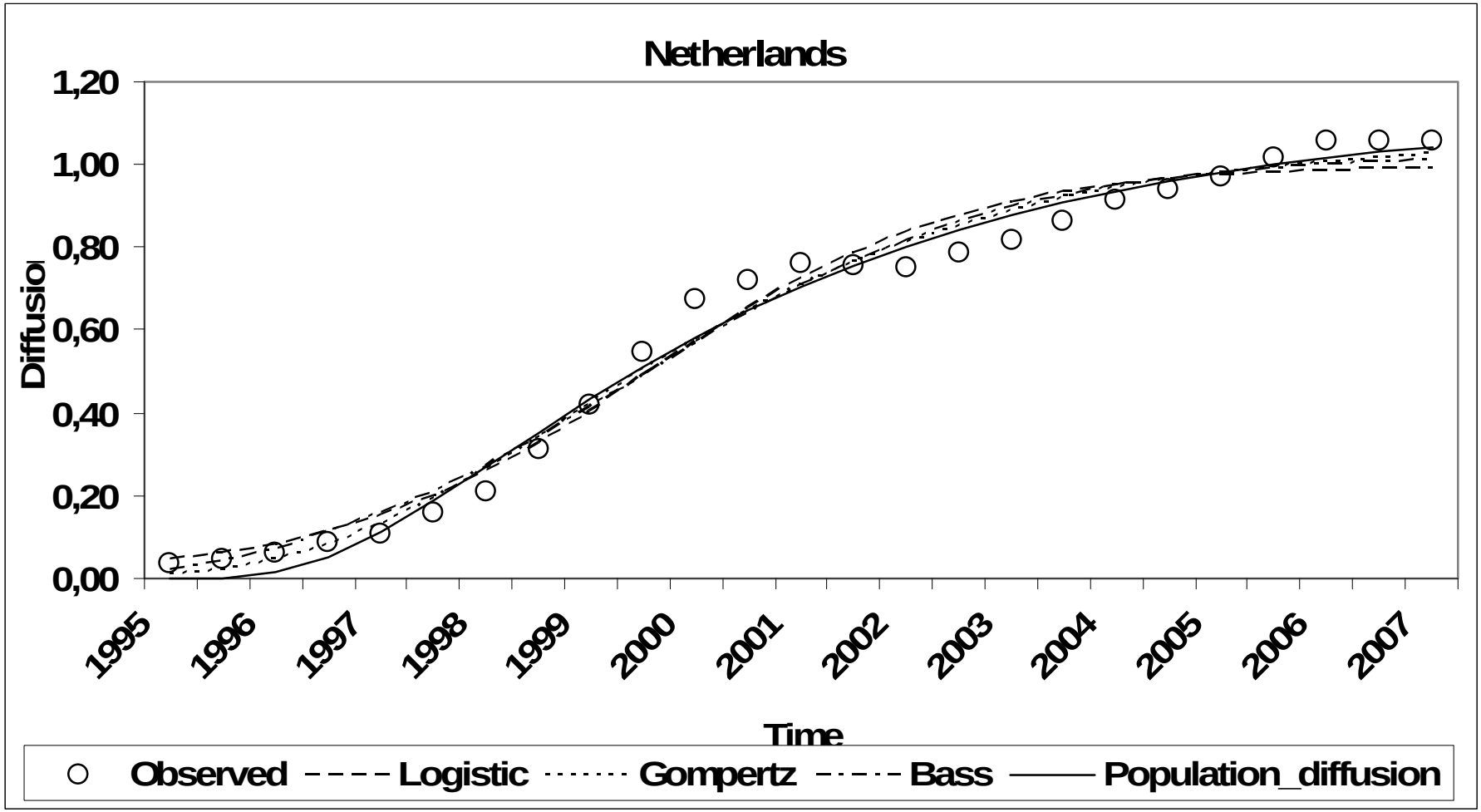
$$N = K e^{\ln\left(a+\frac{bP}{K}\right)+\frac{bP}{aK+bP}\ln\left(\frac{N_0}{K}\right)\left(e^{-r\ln\left(a+\frac{bP}{K}\right)t}-1\right)}$$

$$= \frac{\ln\left(a+\frac{bP}{K}\right)\ln\left(\frac{N_0}{K}\right)e^{-r\ln\left(a+\frac{bP}{K}\right)t}}{}$$

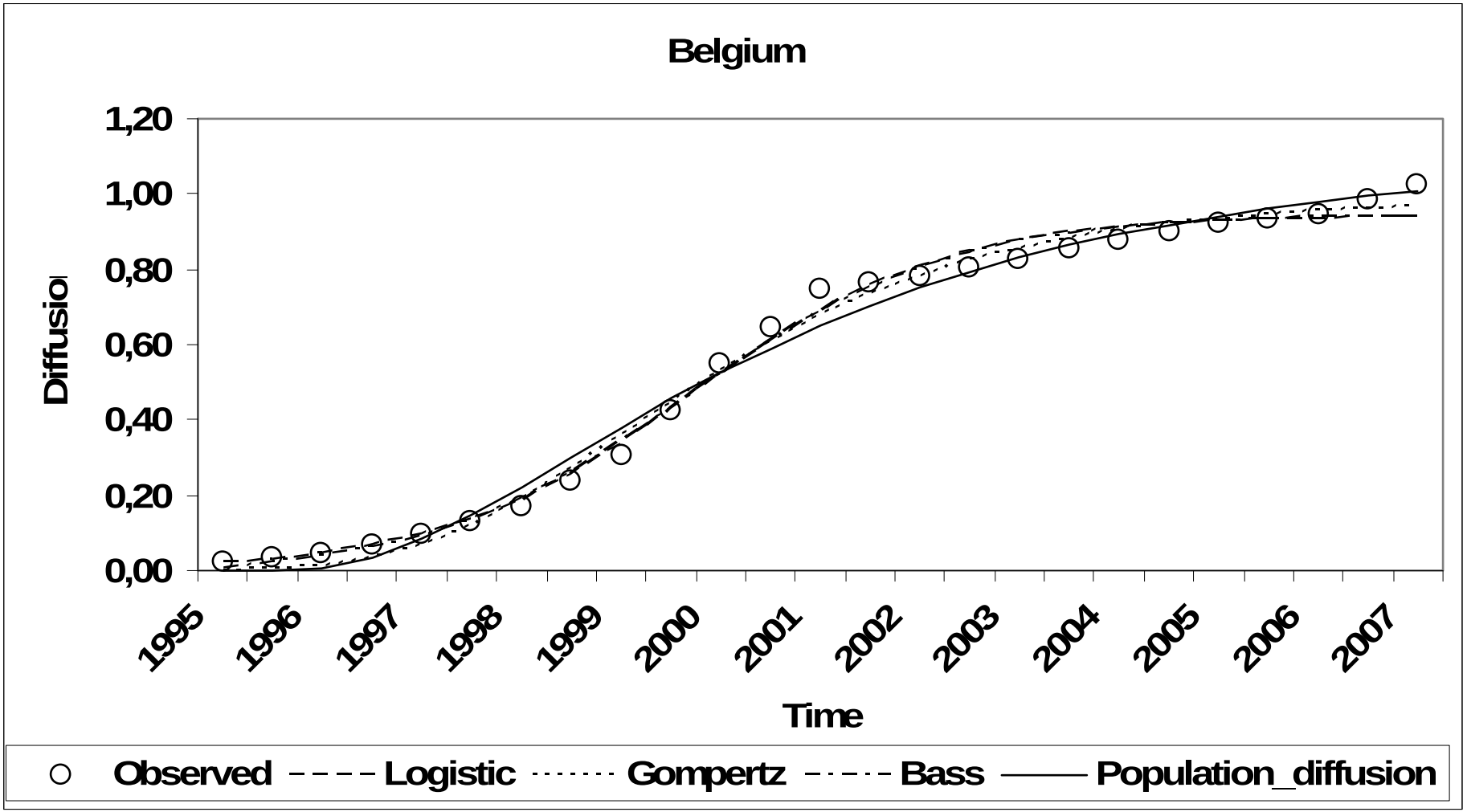
Model evaluation (1) over whole dataset



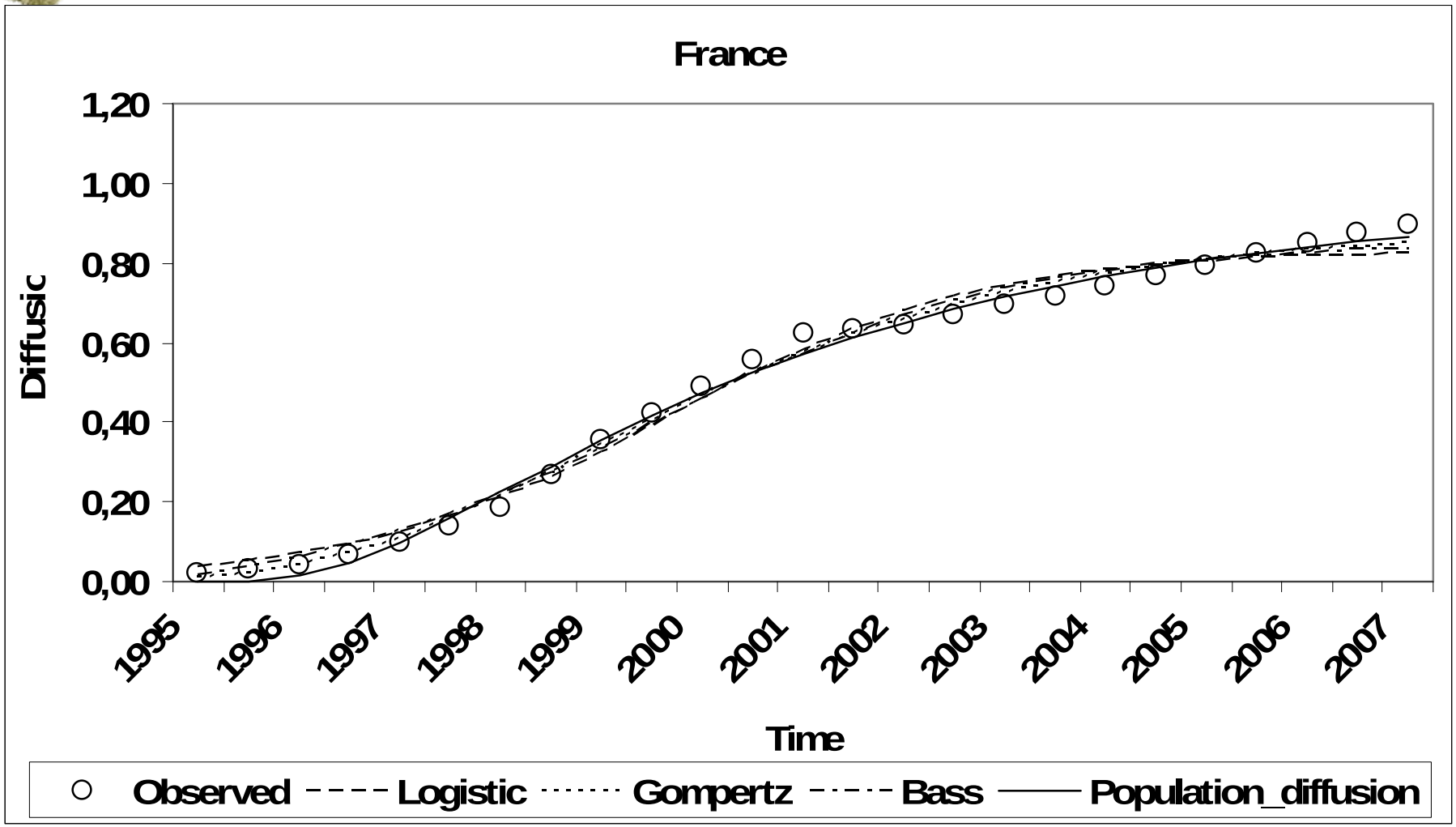
Model evaluation (2)



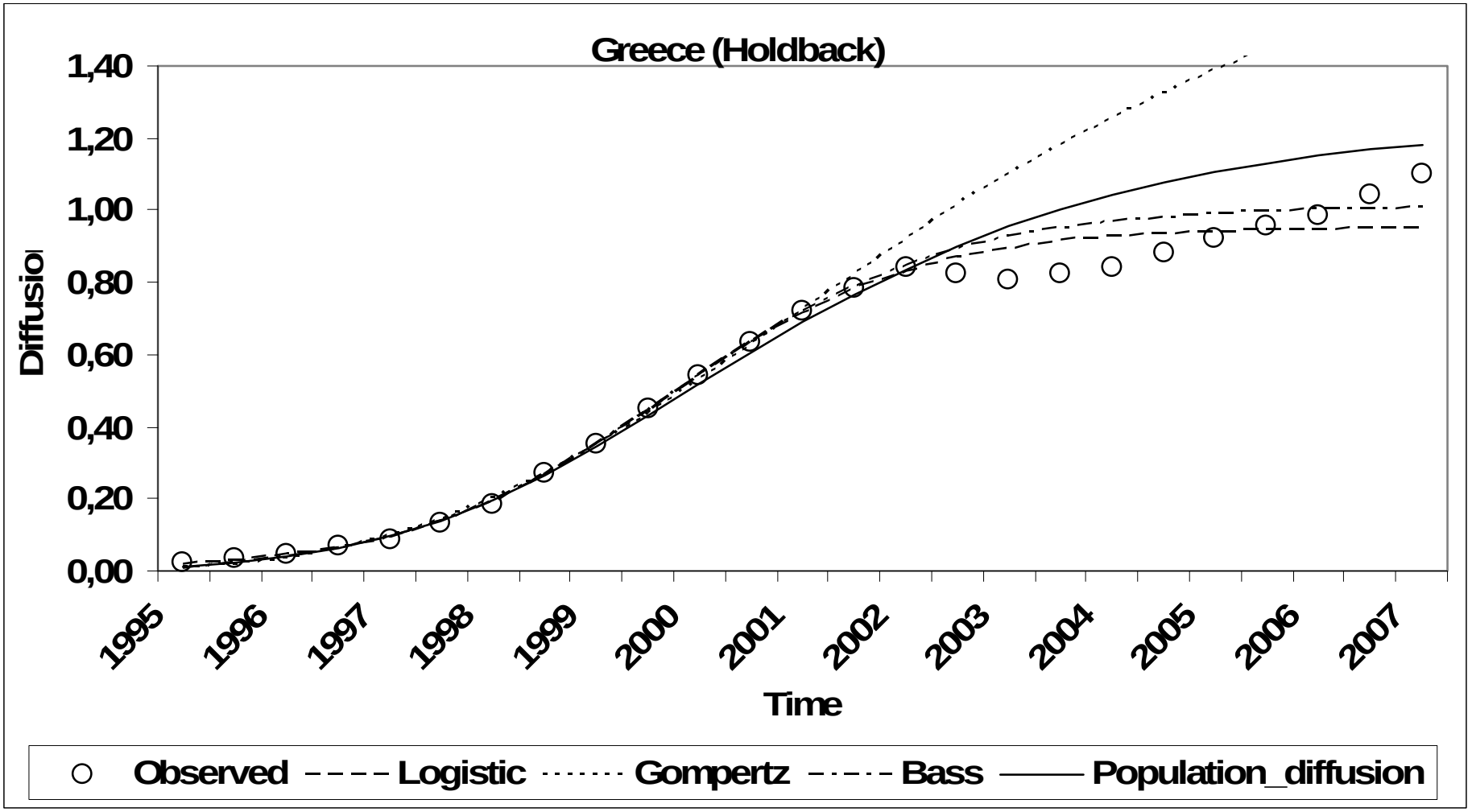
Model evaluation (3)



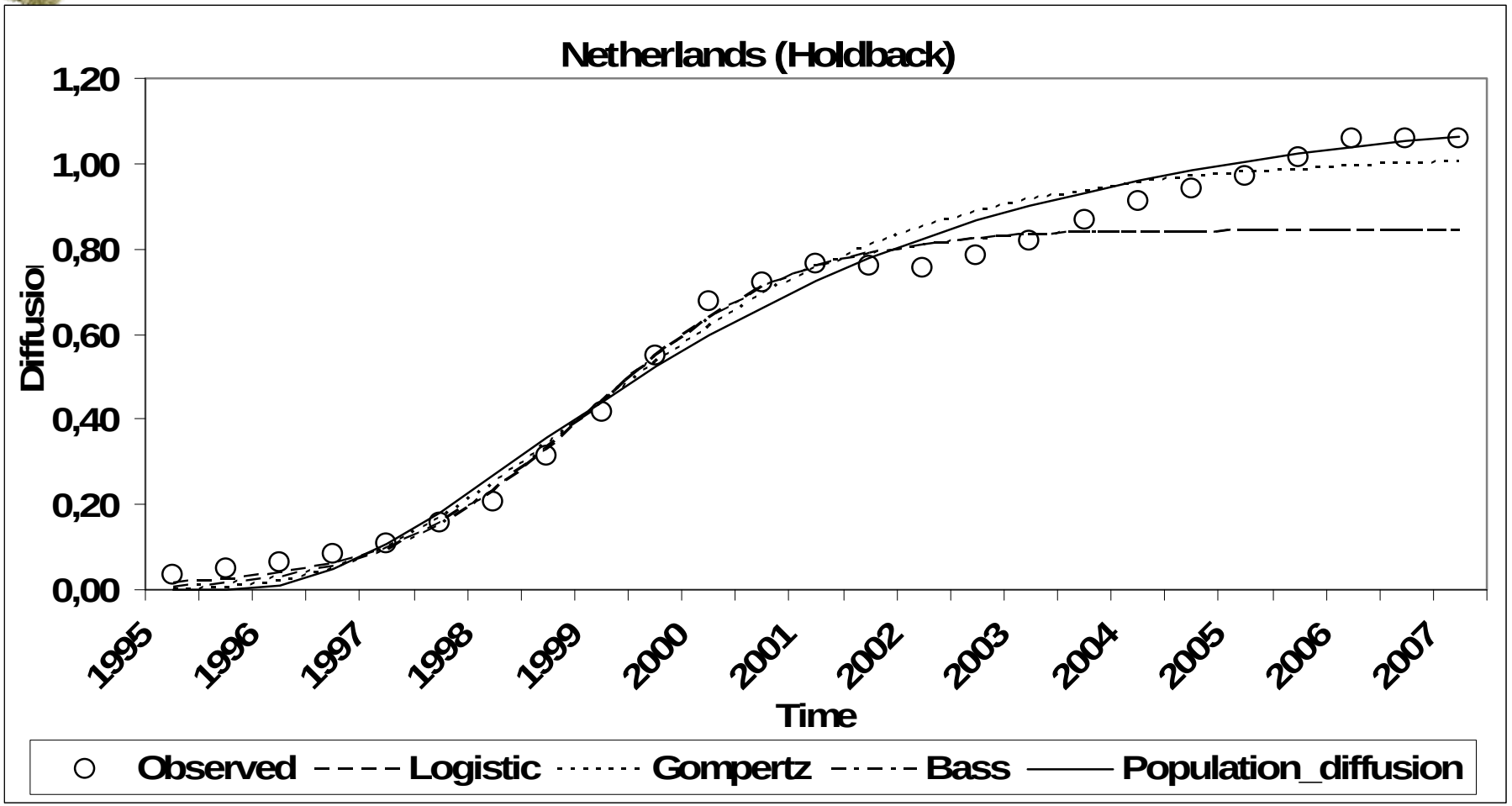
Model evaluation (4)



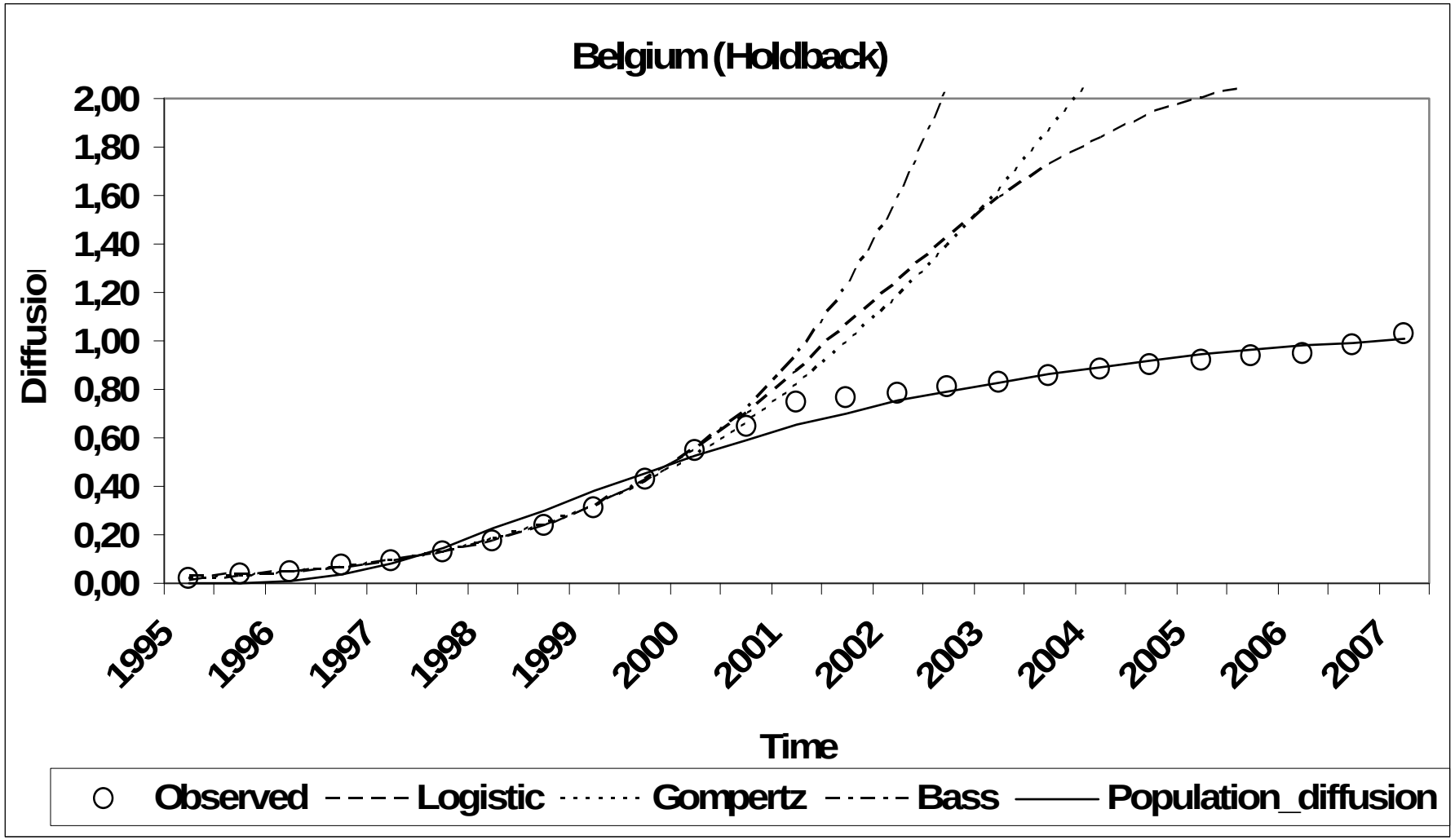
Model forecasting ability(1)
(using hold back sample, up to one year before inflection point)



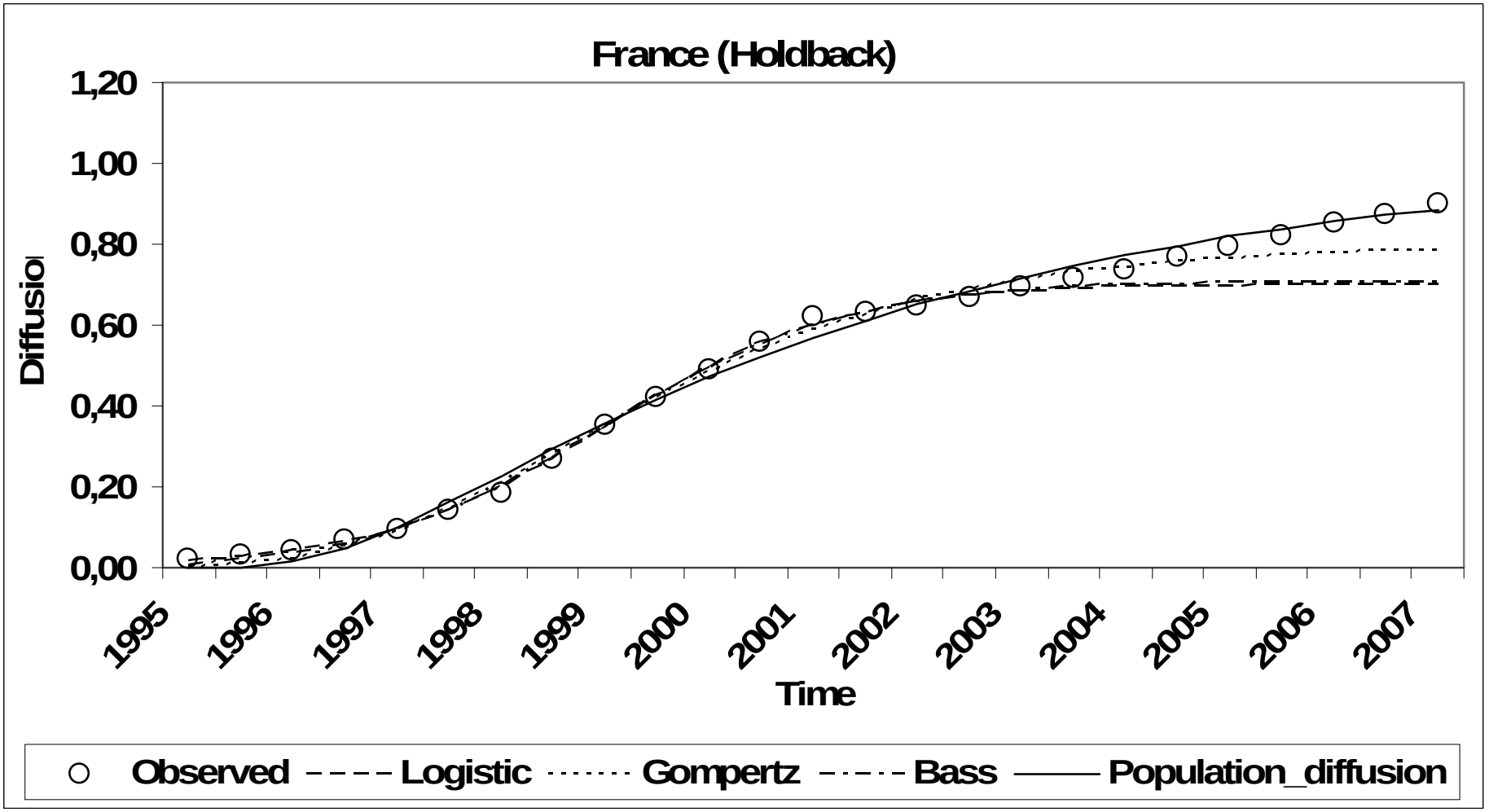
Model forecasting ability(2)



Model forecasting ability(3)



Model forecasting ability(4)



Stochastic realization of PDM (1)

- **The need for stochastic analysis**
  - The rapidly changing environmental socio economic factors, which affect the diffusion's characteristics by adding randomness on the adoption pattern
  - Stochastic models are capable of quantifying uncertainty, caused by both internal to the system, or external factors,
  - Provide a set of possible situations of the process, at each point of time.
  - A deterministic model can be it cannot include all the factors that possibly affect the process and since many of the external parameters are random by their nature, they cannot be accurately estimated and used for forecasting purposes

Stochastic realization of PDM (2)

- **Addition of a stochastic term – Wiener Process**
  - Non-differentiable random function of time t, obtained by sampling the normal probability density:

$$\frac{1}{\sqrt{2\pi t}} e^{-W_t^2 / 2t}$$

- **General formulation**

$$dN = f(N,t)dt + g(N,t)dW(t)$$

Stochastic realization of PDM (3)

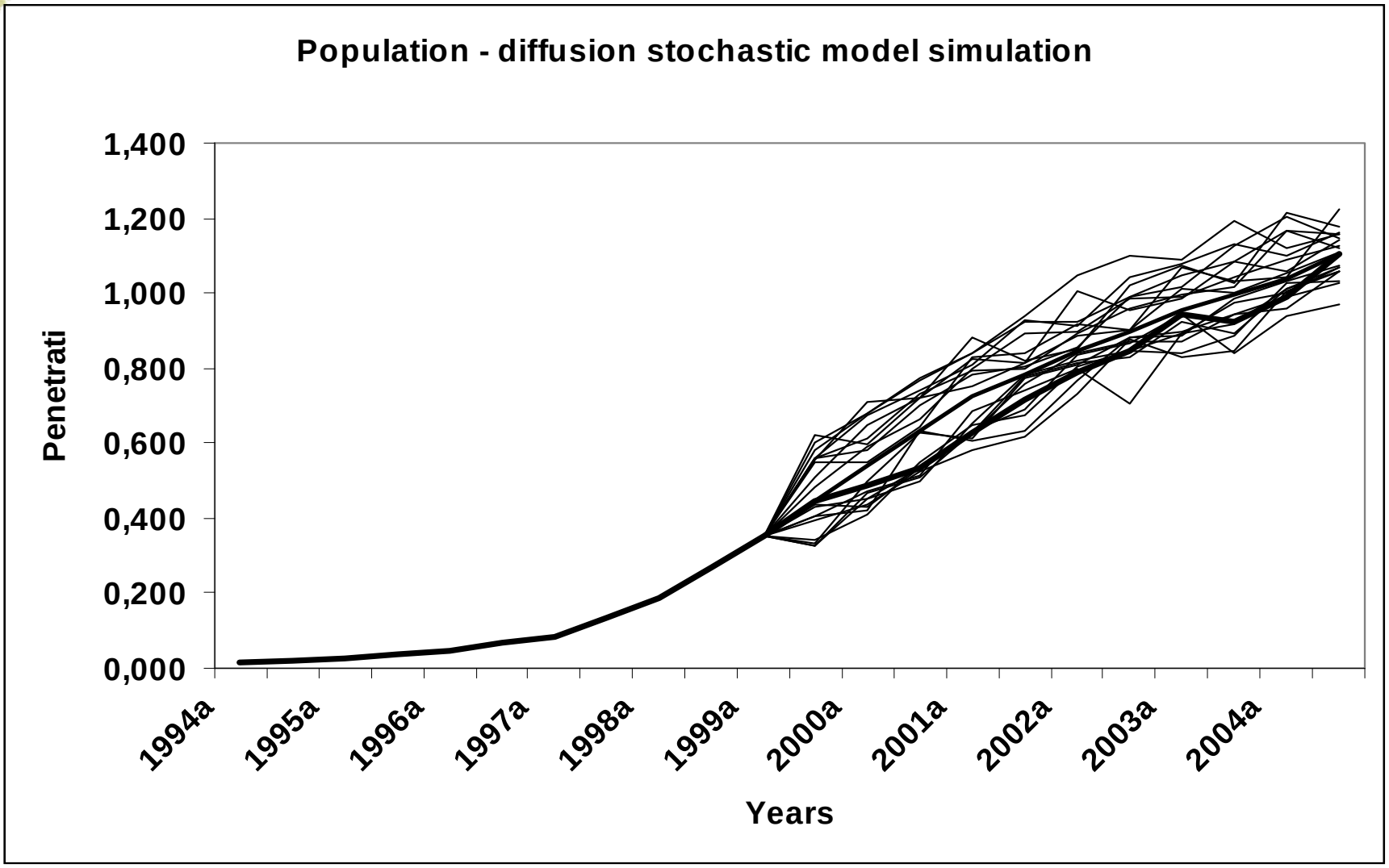
- **PDM formulation**

$$dN = rN\,ln(a + b\frac{P}{N})\left[ln(\frac{K}{N})dt + gdW(t) \right]$$

- **or, equivalently**

$$dN = rN\,ln(a + b\frac{P}{N})ln(\frac{K}{N})dt + rN\,ln(a + b\frac{P}{N})gdW(t)$$

Results of stochastic PDM



Extensions

- **Study the effect of other decision variables, such as**
  - price,
  - advertizing,
  - generation substitution

  over the diffusion process

Thank you!!

Any questions?

Christos Michalakelis