Price War in Heterogeneous Wireless Networks

Patrick Maillé, Bruno Tuffin

Telecom Bretagne and INRIA-Centre Bretagne Atlantique
Rennes, France

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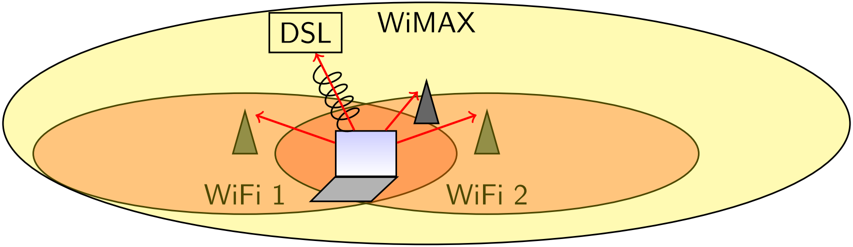
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WiMAX
WiFi 1
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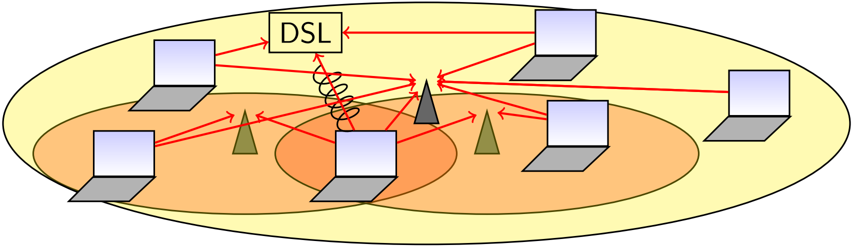
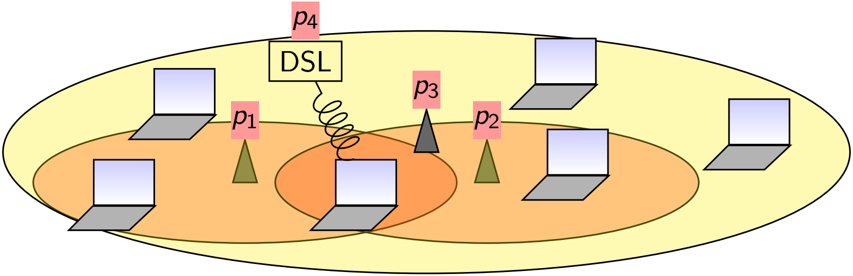


- Interactions among non-cooperative consumers: *game*
- Congested networks provide poorer quality (packet losses)

But **providers** play first!



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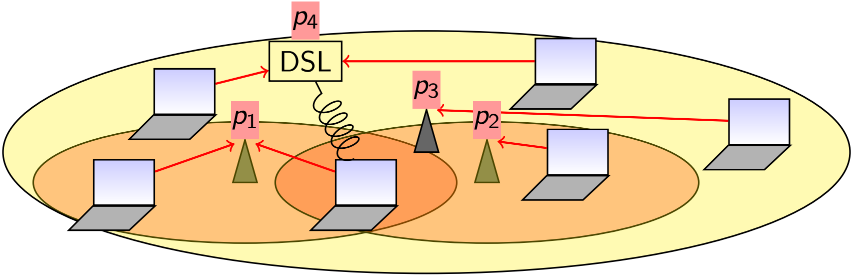

This work: study of the two-level noncooperative game.

1. *Higher level*: **providers** set prices to maximize revenue
2. *Lower level*: **consumers** choose their provider

Related work

Many references on network pricing, with different objectives:

- control congestion, — Key & Massoulié'99, Lazar & Semret'99
- ensure fairness, — Kelly *et al.*'98, Marbach'02
- manage different QoS levels, — Cocchi *et al.*'93, Odlyzko'99
- maximize network revenue. — Paschalidis & Tsitsiklis'00

But only few considering competition among providers:

- wireless providers playing on trans. power — Felegyhazi & Hubaux'06
- studies of peering agreements — He & Walrand'03'05, Shakkotai & Srikant'05
- competition with delay-sensitive users — Acemoglu & Ozdaglar'06, Hayrapetyan *et al.*'06

This work: competition among providers with **loss-sensitive users** and **minimal regulation** $\Rightarrow$ performance of the outcome?

Outline

Competition model

- Infinitely small users (price-takers)
- N competing providers declaring price and capacity ($\mathcal{I} := \{1, \ldots, N\}$)
- Two simplified cases for the topology:

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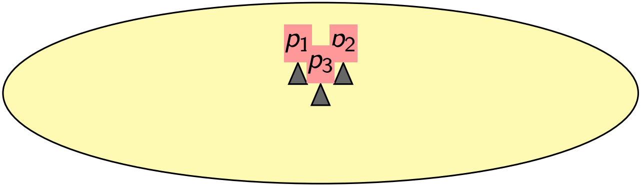

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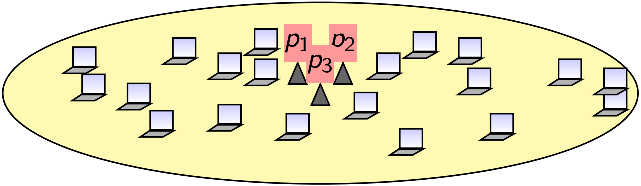

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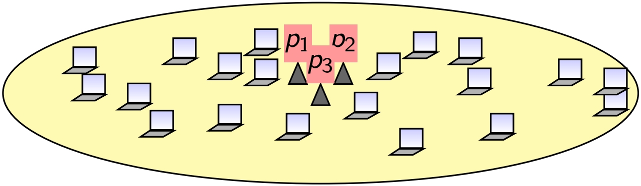
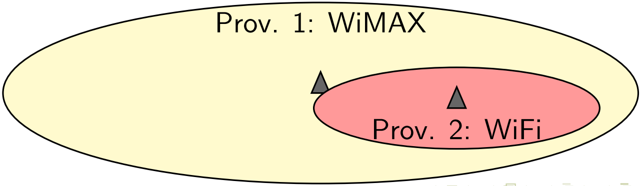


2. $N = 2$, one coverage area included in the other



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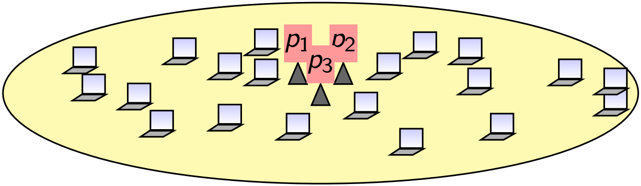

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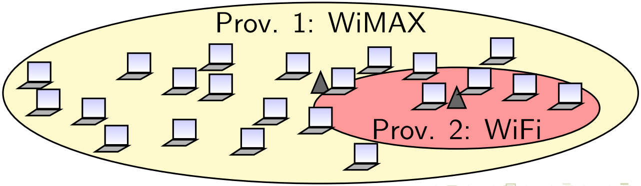
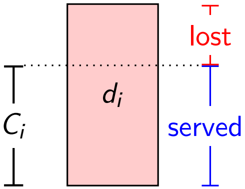


Communication model: Packet losses

- Time is slotted
- Each provider i has finite capacity C_i
- If total demand d_i at provider i exceeds C_i: exceeding packets are *randomly* lost



$$\mathbb{P}(\text{successful transmission}) = \min\left(1, \frac{C_i}{d_i}\right)$$

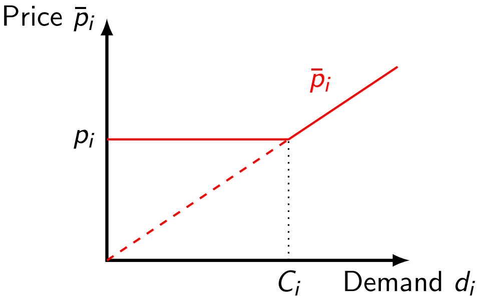
$$\Rightarrow \text{Expected number of transmissions} = \frac{1}{\mathbb{P}(\text{success})} = \max\left(1, \frac{d_i}{C_i}\right)$$

Only "regulation": pay for what you send

The price p_i at each provider i is per packet *sent* Marbach'02

$\Rightarrow$ If several transmissions are needed, the user pays several times

$$\bar{p}_i := \textit{perceived} \text{ price at } i = \mathbb{E}[\text{price per packet}] = p_i \max\left(1, \frac{d_i}{C_i}\right)$$



Analysis of user choices: Wardrop equilibrium

- Users choose the provider(s) i with lowest $\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$

$\Rightarrow$ For a given coverage zone Z, all providers with customers from that zone end up with the same perceived price $\bar{p}_i = \bar{p}_z$ Wardrop'52

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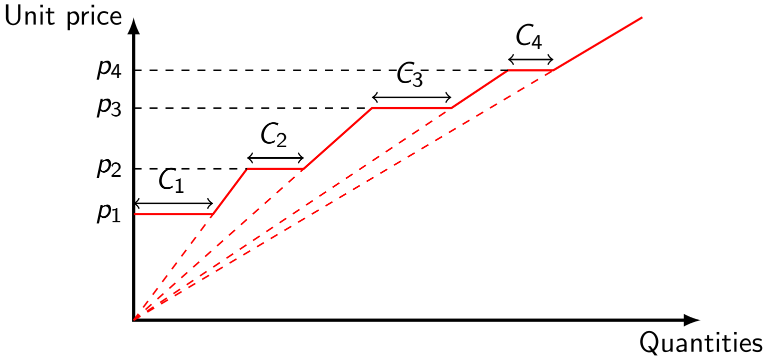
- The total demand level in a zone z depends on that price:

$$d_z = \alpha_z D(\bar{p}_z), \quad i.e. \quad \bar{p}_z = \underbrace{v}_{\text{marg. val. function}}\left(\frac{\sum d_{i,z}}{\alpha_z}\right)$$

with D the total demand function, α_z the population proportion in zone z, and $d_{i,z}$ the demand in zone z for provider i.

Wardrop equilibrium: illustration

1. Case of common coverage area



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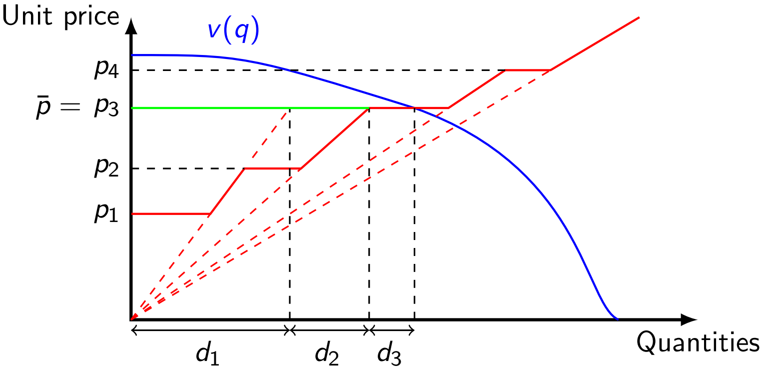

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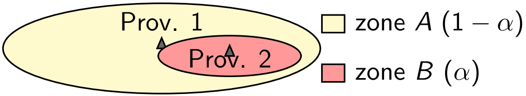
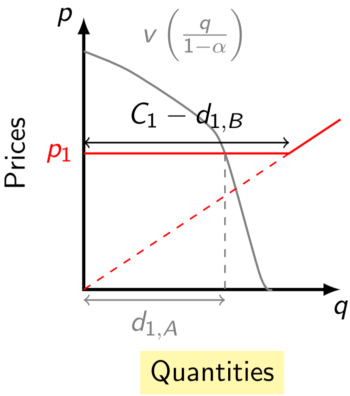
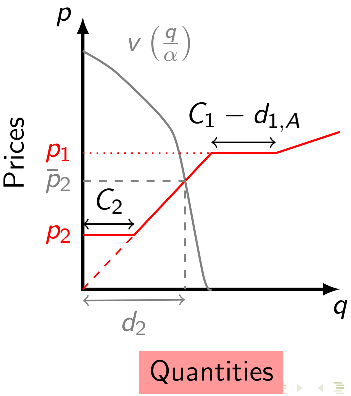
2. Case of two providers







Wardrop equilibrium: illustration

1. Case of common coverage area

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Mathematical formulation

For each zone z and each provider i, j, at Wardrop equilibrium

$$\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$$

$$d_z = \alpha_z D\left(\min_{i \in z} \bar{p}_i\right)$$

If $i, j \in z$, then $\bar{p}_i > \bar{p}_j \Rightarrow d_{i,z} = 0$.

Wardrop equilibrium: existence and uniqueness

Proposition

For all price profile, there exists at least a Wardrop equilibrium. Moreover, the corresponding perceived prices of each provider are unique.

NB: demand repartition among providers is not necessarily unique.

Higher level: price competition game

- Providers set their price p_i *anticipating users reaction*
  $\Rightarrow$ Providers are Stackelberg leaders
- Provider i's objective: $R_i := p_i d_i$.

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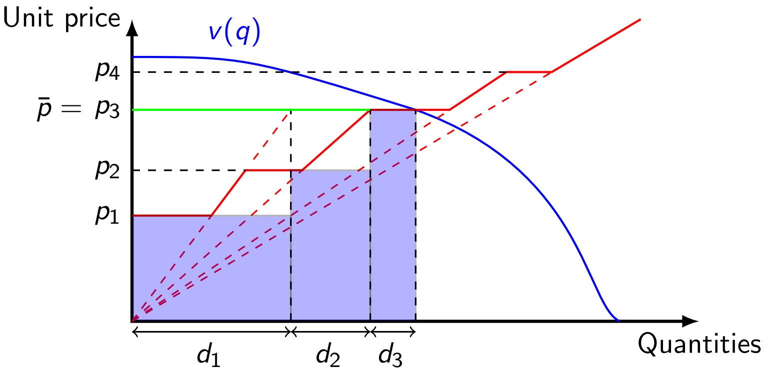


Price competition, main results (1/2)

Case of a common coverage area

Proposition

Under condition (1) or (2), there exists a **unique Nash equilibrium** *on price war among providers, given by*

$$\forall i \in \mathcal{I}, \quad \begin{cases} p_i & = & v\left(\sum_{j\in\mathcal{I}} C_j\right) \\ d_i & = & C_i. \end{cases}$$

- *Sufficient conditions:*

$$\text{For each provider } i, \quad C_i \leq \sum_{j\neq i} C_j \tag{1}$$

$$-\frac{D'(p)p}{D(p)} > 1, \quad \forall p \tag{2}$$

Proof sketch

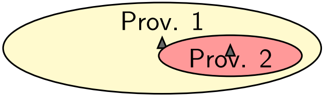
1. First step: $\left\{ \begin{array}{lcl} p_i & = & v\left(\sum_{j\in\mathcal{I}} C_j\right) \\ d_i & = & C_i \end{array} \right.$ defines a Nash equilibrium; i.e. no provider can improve its benefit by changing his price.
2. Second step: no other point can be a Nash equilibrium.
   - We partition the provider set $\mathcal{I}$ into three subsets:

$$\begin{array}{lcl} \mathcal{I}_s & := & \{i \in \mathcal{I} : d_i > C_i\}, \\ \mathcal{I}_0 & := & \{i \in \mathcal{I} : d_i = C_i\}, \\ \mathcal{I}_u & := & \{i \in \mathcal{I} : d_i < C_i\}. \end{array}$$

   - First, $\mathcal{I}_s = \emptyset$. If not, provider $i_s \in \mathcal{I}_s$ increasing (alone) its unit price p_{i_s} to $p^n_{i_s} = p_{i_s} + \varepsilon$, improves its revenue.
   - Assume then $\mathcal{I}_u \neq \emptyset$. Using the fact that $\mathcal{I}_s = \emptyset$, $i_u \in \mathcal{I}_u$ has interest in decreasing (alone) its price: its revenue would increase (due to increasing demand).

Price competition, main results (2/2)

Case of 2 providers



Proposition

Under Condition (2), there exists a unique Nash equilibrium (p_1^*, p_2^*) in the price war between providers. The Nash equilibrium is characterized as follows.

- *If $\frac{C_1}{1-\alpha} \leq \frac{C_2}{\alpha}$, then $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. Zone B is left to provider 2 by provider 1.*
- *If $\frac{C_1}{1-\alpha} > \frac{C_2}{\alpha}$ then $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. Zone B is shared by the providers.*

Condition (2):

$$-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$$

Social Welfare considerations

- A performance measure of the outcome $(d_1, ..., d_I)$ of the game = overall value of the system

$$\text{Social Welfare} := \sum_{\text{zones } Z} \underbrace{\int_0^{\text{throughput}_z} v\left(x/\alpha_z\right) dx}_{\text{users willingness-to-pay}},$$

with $\text{throughput}_z = \sum_i d_{i,z} \min\left(1, \frac{C_i}{d_i}\right)$.

- **Remark:** under the conditions of previous Propositions, the Social Welfare maximization problem leads to the same outcome as the price war.
- **Consequence:** The Nash equilibrium corresponds to the socially optimal situation: the Price of Anarchy is 1!.

Conclusions and perspectives

We have analyzed a pricing game among providers

- Characterized how demand is split (following Wardrop's principle),
- studied the Nash equilibrium of the pricing game (characterization, uniqueness),
- shown that its outcome is socially optimal,

Perspectives

- What happens if providers partially share their capacities?
- What if providers play on capacities along with prices?
- What about the multiclass case?
- What about the dynamics of the model? How to drive to the equilibrium?
- What about more complicated topologies?