Sponsored Search Engines in Competition: Advertisers Behavior and Engines Optimal Ranking Strategies

Patrick Maillé and Bruno Tuffin

Telecom Bretagne and INRIA Rennes - Centre Bretagne Atlantique

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Outline

1. Introduction to adword auctions & goal of the work
2. Model
3. Average prices and winning probabilities at search engines
4. Game between advertisers
5. Which mechanism to implement at the SE level?
6. Conclusions/Future activities

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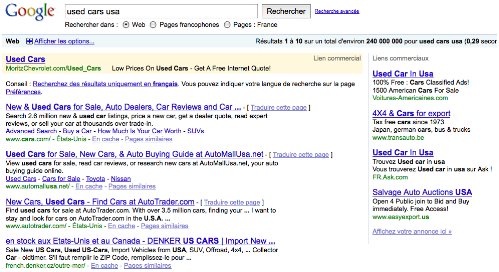
Introduction to adword auctions

- Search engines play a crucial role in the Internet.
- Revenue through advertising slots, usually displayed at the top or right of the search page.
- Advertisers submit bids for relevant keywords only.



- Allocation of slots thanks to adword auctions.
  - combined revenue of Yahoo! and Google in 2005: \$11 billion in 2005
  - expected to count for 40% of total advertising revenue.

Auction principle (single keyword, K slots)

- Advertisers submit bids for specific keywords.
- Each time there is a search on that keyword:
  - advertisers are ranked and allocated slots according to a prespecified criterion:
    - bid value (initially for Yahoo!)
    - the revenue they will generate (more or less Google).
  - Possible payment rules:
    - *Pay-Per-Impression* (PPI): advertisers charged every time their ad is displayed
    - *Pay-Per-Click* (PPC): advertisers is charged only when the ad is clicked
    - *Pay-Per-Transaction* (PPT): advertisers charged when a sell.
  - Amount to be paid each time?
    - First Price: advertisers pay their bid
    - Generalized Second Price (GSP): they pay the bid of advertiser below them in the ranking
    - Vickrey-Clarke-Groves (VCG) auctions: you pay the opportunity cost that your presence introduce to all other advertisers.
- In use: PPC and GSP. But bid-based or revenue-based ranking?

Goal of our work

- Almost all works deal with monopolistic search engines,
- But those engines are in competition
  $\Rightarrow$ What is the best ranking strategy given this competition?
- Non-cooperative game between search engines.
- Two-levels game:
  - Largest time scale: search egines choose their ranking strategy (maximizing) their revenue
  - Smallest time scale: advertisers in competition for the advertising slots (by splitting their avertisement budget over the engines)
- Game played by backward induction.
- Requires to compute the average prices and winning probabilities at search engines depending on the strategies of all actors.

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Basic model

- Two search engines (SE), labelled 1 and 2
  - a single advertisement slot at each SE
  - considering a single keyword, with λ, average number of searches per unit of time
  - α: (fixed) market share of SE 1 ($\alpha\lambda$ searches on SE 1).
  - Both SE apply GSP charging scheme.
- k advertisers:
  - budget b, taken from cdf $G(b)$
  - valuation per click v, taken from the cdf $F(v)$
  - Click-Through-Rate (CTR) *separable*, as the product of the CTR of the search engine, q_1 and q_2 for SE 1 and 2 respectively, and of the CTR c_i of the considered Advertiser i.
  - Goal of advertiser i: split their budget b_i over the two SEs: β_i on SE 1 and $1 - \beta_i$ on SE 2.
  - Remark: advertisers' interest is to bid their valuation v_i since GSP is VCG when a single slot.

Key parameters

- $w_j(v_i)$: probability that i wins on SE j given that her bid/valuation is v_i
- $\mathbb{E}[R_j|v]$ being the average price paid on SE j *having won with* v.

Rates of advertisers:

$$\lambda_i^{(1)} = \min\left(\frac{\beta_i b_i}{q_1 c_i \mathbb{E}[R_1|v_i] w_1(v_i)}, \alpha\lambda\right)$$
$$\lambda_i^{(2)} = \min\left(\frac{(1-\beta_i) b_i}{q_2 c_i \mathbb{E}[R_2|v_i] w_2(v_i)}, (1-\alpha)\lambda\right).$$

First term: number of times Advertiser i can bid in order to reach her budget repartition, in average.

Probability that i bids on 1 and 2:

$$p_i^{(1)} = \min\left(\frac{\beta_i b}{\alpha\lambda q_1 c_i \mathbb{E}[R_1|v_i] w_1(v_i)}, 1\right)$$
$$p_i^{(2)} = \min\left(\frac{(1-\beta_i) b}{(1-\alpha)\lambda q_2 c_i \mathbb{E}[R_2|v_i] w_2(v_i)}, 1\right).$$

Revenues, for $\beta = (\beta_1, \ldots, \beta_k)$ profile of strategies of advertisers:

$$U_i(\beta) = q_1 c_i w_1(v_i)\lambda_i^{(1)}(v_i - \mathbb{E}[R_1|v_i]) q_2 c_i w_2(v_i)\lambda_i^{(2)}(v_i - \mathbb{E}[R_2|v_i]).$$

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With bid-based ranking

If $\pi_\ell = p_\ell$ if ℓ bids, $1 - p_\ell$ otherwise:

- Probability to win with valuation v: probability the $k-1$ other have valuation less than v

$$w(v) = \sum_{j_1,\dots,j_{k-1}\in\{0,1\}} (F(v))^{(\sum_{l=1}^{k-1} j_l)} \left(\prod_{\ell=1}^{k-1} \pi_\ell\right)$$

A priori expected probability of an advertiser with valuation v.

- $\mathbb{E}[R|v]$: expected value of the second largest bid, taking into account the probabilities that each bidder submits a bid:

$$\sum_{j_1,\dots,j_{k-1}\in\{0,1\}} \left(\prod_{\ell=1}^{k-1} \pi_\ell\right) \int_{u\le v} \frac{(\sum_{l=1}^{k-1} j_l) u (F(u))^{(\sum_{l=1}^{k-1} j_l)-1} f(u)}{(F(v))^{(\sum_{l=1}^{k-1} j_l)}} du.$$

Simplified expressions symmetric advertisers, $k = 2$ and/or specific distributions (see the proceedings).

$\Rightarrow$ the values of p_i, $\mathbb{E}[R|v_i]$ and $w(v_i)$ can be derived.

With revenue-based ranking

- Probability to win with valuation v and CTR cq: her product qcv larger than the $qc_\ell V_\ell$ of each other bidding advertiser ℓ

$$w(v) = \sum_{j_1,\ldots,j_{k-1}\in\{0,1\}} \prod_{\ell=1}^{k-1} \left(F\left(v\frac{c}{c_\ell}\right)\right)^{j_\ell} \pi_\ell.$$

- Revenue cdf of ℓ: $\mathbb{P}[qc_\ell V \leq x] = F(x/(qc_\ell))$. Integrating the (condittional) density of the second highest revenue, we get $cq\mathbb{E}[R|v]$:

$$\sum_{j_1,\ldots,j_{k-1}\in\{0,1\}|} \left(\prod_{\ell=1}^{k-1} \pi_\ell\right) \int_{x\leq vqc} \frac{\sum_{\ell:j_\ell=1} x\frac{1}{qc_\ell} \prod_{l=1;l\neq\ell}^{k-1} (F(x/(qc_l)))^{j_l} f(x/(qc_\ell))}{\prod_{\ell=1}^{k-1}\left(F\left(v\frac{c}{c_\ell}\right)\right)^{j_\ell}} dx.$$

$\Rightarrow$ The values of p_i, $\mathbb{E}[R|v_i]$ and $w(v_i)$ can be derived too.

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Game between advertisers on budget repartition at each SE

We consider two SEs and two advertisers.

- The two advertisers play, trying to (selfishly) maximize their own utility/revenue.
- Equilibrium notion, Nash equilibrium: profile of proportion strategies (β_1^*, β_2^*) st $\forall \beta_1, \beta_2 \in [0,1]$,

$$U_1(\beta_1^*, \beta_2^*) \geq U_1(\beta_1, \beta_2^*) \text{ and } U_2(\beta_1^*, \beta_2^*) \geq U_2(\beta_1^*, \beta_2).$$

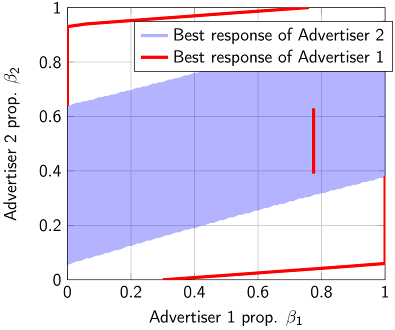
- To determine the Nash equilibria (if any), we define the *best response* of each advertiser as a function of the strategy of its opponent:

$$\begin{aligned} \mathrm{BR}_1(\beta_2) &= \arg\max_{\beta \in [0,1]} U_1(\beta, \beta_2) \text{ and} \\ \mathrm{BR}_2(\beta_1) &= \arg\max_{\beta \in [0,1]} U_2(\beta_1, \beta). \end{aligned}$$

- Graphically, the set of Nash equilibria is the (possibly empty) set of intersection points of BR curves.

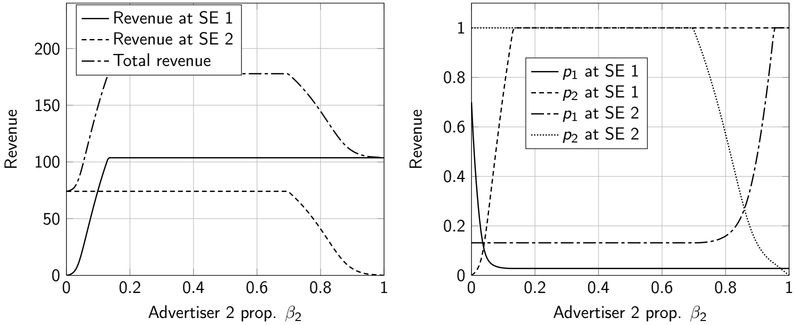
Illustration: both SEs implement bid-based pricing

$V \equiv U[0,20]$, $\alpha = 0.6$, $\lambda = 100$, $q_1 = 0.5$, $q_2 = 0.6$, $b_1 = 5$, $c_1 = 0.5$, $b_2 = 20$, $c_2 = 0.4$, $v_1 = 10$, $v_2 = 9$ and $p_r = 0.1$



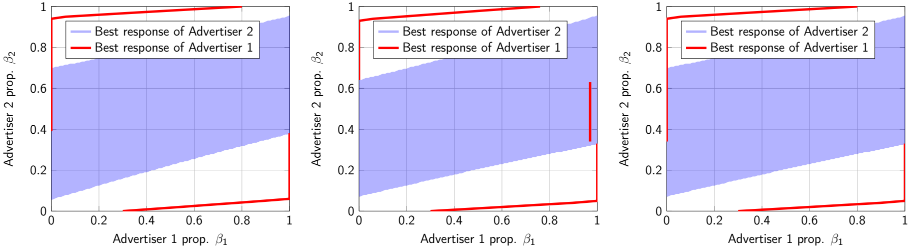
- for each fixed β_1, there is actually an interval for the best response of Advertiser 2 (blue)
- best response of Advertiser 1 in terms of β_2, we obtain the red curve
- set of Nash equilibria: $\{0.775\} \times [0.39, 0.63]$.

Explanation of an interval as best response, for $\beta_1 = 0.21$



- Left: revenue indeed constant
- Right: bidding probability constant
- The probability of bidding is maximal, equal to 1, independent of the submitted budget $\Rightarrow$ the budget is not fully spent.

3 other cases of ranking possibilities: bid based- revenue based (B-R), R-B and R-R



Nash equilibria:

- For the B-B case, all the profiles $(\beta_1, \beta_2) \in \{0.775\} \times [0.39, 0.63]$;
- for the B-R case, it is $\{0\} \times [0.39, 0.695]$;
- for the R-B case, $\{0.97\} \times [0.34, 0.63]$;
- for the R-R case, $\{0\} \times [0.34, 0.695]$.

Remark: when SE 2 implements revenue-based ranking, Advertiser 1's strategy at a Nash equilibrium is to put all her budget on SE 2.

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Game between SEs on the ranking strategy

Anticipating the advertisers' decisions, the SEs seek to maximize their revenues from advertisement.

Another level of game: $(\text{Rev}_1, \text{Rev}_2)$ in terms of the rules used by SE 1 (line) and SE2 (column)

	B	R
B	(10.15, 3.62)	(1.20, 11.06)
R	(11.32, 1.32)	(1.50, 11.06)

- Best responses in red. Nash equilibrium: both elements in red.
- R-R: Nash equilibrium: in agreement with Yahoo!'s move to follow Google.
- For other sets of parameters such that B-R is an equilibrium.
  $\Rightarrow$ close look necessary for SEs!

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We have

- defined a model describing two search engines in competition for advertisers in adword auctions
- derived how advertisers should behave depending on the ranking strategies of engines
- llustrated how engines can (competitively) play on the ranking strategy.

Future activities:

- extend our study to the situation where SEs propose more than one slot
- look more closely at Google against Yahoo!