COST 605

Econ@Tel
A Telecommunications Economic COST Network

WG4 meeting

15 February 2011

Exploring pricing rules in spectrum auctions

José Luis Gómez-Barroso
UNED – Universidad Nacional de Educación a Distancia

Index

- Introduction
- The auction
- Pricing rules
- Scenarios
- Agent-based model
- Computational Experiments
- Concluding remarks

Introduction

- The radio airwave spectrum is a scarce resource for the use of which licenses are allocated by governments.
- Variables in the design of an auction
  - Open vs. sealed bid
  - Pricing rules
  - Sequential vs. simultaneous auctions
  - Individual lots vs. packaged lots
  - Other variables

Introduction

- We have simulated the sale of a portion of the *digital dividend* that could be representative of any European country of medium to large size (different degrees of asymmetries among participants have been tested).
- Lots can be complements (*exposure problem*) and substitutes: combinatorial auctions the best way to allocate.
- Which is the best pricing rule to obtain efficient allocation and revenues maximization?.
- Analysis of : 1st, VCG and BPO mechanisms.
- Co-evolutionary system to simulate bidding behavior.

+ The auction

- Combinatorial single round sealed-bid auction
  - N lots: $I=(1, 2, \ldots N)$
  - Combination of lots: S
  - M bidders: $J=(1, 2, \ldots M)$
  - Valuation of participant j for combination S: $v_j=(S)$
  - Bidders are allowed to submit in one single round as many bids as they wish for any combination of available lots: $b_j=(S)$
  - XOR bidding language: winning bidders can only win a single bid (bids are mutually exclusive).

+ The auction

- Combinatorial single round sealed-bid auction
  - Given all bids submitted, the auctioneer determines the combination of feasible bids that maximizes his revenues, i.e., solves the Winner Determination Problem.

$$\max \sum_{j\in J} \sum_{S\subseteq I} b_j(S) x_j(S)$$

subject to

$$\sum_{S\supseteq\{i\}} \sum_{j\in J} x_j(S) \le 1 \forall i \in I,$$

$$\sum_{S\subseteq I} x_j(S) \le 1 \forall j \in J,$$

$$x_j(S) \in \{0,1\} \forall S \subseteq I, \forall j \in J.$$

→ Winning bidders $j \in W$ → Price to pay?

- The WDP is a NP-complete problem (Sandholm, 2002). We used an A* based on branch on bids (BOB) formulation search algorithm (Saez et al., 2008).

Pricing rules: First price

- First-price mechanism
  - Each participant will pay an amount equal to the bid made for the combination of lots that has won (S^*).

$$\pi_j^{1st} = b_j(S_j^*)$$

  - Seller's final income is equal to the sum of the payment of all winning bidders.

$$\theta^{1st} = \sum_{j \in W} \pi^{1st}{}_j$$

  - Advantages: simple to implement, reduces the risk of collusion among participants while fostering their participation.
  - Drawbacks: participants tend to shade their bids, a strategy that affects the final allocation of the lots.

Pricing rules: First price

Table of bids

	$b_1(S)$	$b_2(S)$	$b_3(S)$	$b_4(S)$
A	20	10		10
B		20	10	10
C	10		20	10
AB				28

Winners after solving the WDP → $\theta^{1st} = 60$

Pricing rules: VCG

- Vickrey-Clarke-Groves (VCG) mechanism
  - The price paid by the winning participant is equal to the opportunity cost of the items won. This amount depends only on the bids made by the winning bidder's rivals (Clarke 1971, Groves 1973, and Vickrey 1961).

$$\pi_j^{VCG} = \alpha_j - \sum_{k \neq j} b_k(S_k^*)$$

where $\alpha_j = \max \left\{ \sum_{k \neq j} b_k(S_k) \middle| \sum_{k \neq j} S_k \leq I \right\}$

  - Seller's final income is equal to the sum of the payment of all winning bidders.

$$\theta^{VCG} = \sum_{j \in W} \pi^{VCG}{}_j$$

Pricing rules: VCG

- Vickrey-Clarke-Groves (VCG) mechanism
  - Advantages: incentive compatible (truth telling).
  - Drawbacks: revenues might be to low
    - *result is not in the core*: if there is a coalition of bidders willing to offer another result in which all members of the coalition, including the seller, obtain higher total value.

Pricing rules: VCG

Table of bids

	$b_1(S)$	$b_2(S)$	$b_3(S)$	$b_4(S)$
A	20	10		10
B		20	10	10
C	10		20	10
AB				28

VCG payment for B1 (analogously for B2 and B3):

$$\pi_j^{VCG} = \alpha_j - \sum_{k \neq j} b_k(S_k^*)$$

$$\alpha_1 = b_4(A) + b_2(B) + b_3(C) = 10 + 20 + 20 = 50 \Rightarrow \pi_1^{VCG} = 50 - 40 = 10 \Rightarrow \theta^{VCG} = 30$$

$$\sum_{k \neq 1} b_k(S_k^*) = b_2(B) + b_3(C) = 20 + 20 = 40$$

Pricing rules: BPO

- Bidder-Pareto-optimal (BPO) core mechanism
  - To try to alleviate the problem caused by the previously studied pricing rules, new methods have been developed.
  - The BPO mechanism works off of the same efficient allocation of lots that the VCG mechanism establishes, but imposes restrictions on payments so that they are updated in successive iterations until a result in the core is achieved, Day and Raghavan (2007).

+
Pricing rules: BPO

- Steps to calculate BPO payments
  - Vector of payments π_j^t (the first one: $\pi_j^1 = \pi_j^{VCG}$) $\sum_{j \in W} \pi_j^t$
  - Calculate coalitional contribution
    - Winning bidders: opportunity cost $q_j(S_j, \pi_j^t) = b_j(S_j) - (b_j(S_j^*) - \pi_j^t)$
    - Non winning bidders: initial bid $q_j = b_j(S)$
  - WDP with the coalitional contribution (highest coalitional value $z(\pi^t)$)
    - If there is no blocking coalition the process finishes $\sum_{j \in W} \pi_j^t = z(\pi^t)$
    - If there is a blocking coalition: $\sum_{j \in W} \pi_j^t < z(\pi^t)$

  BPO core payments recalculated applying the core constrains

  $\sum_{j \in W \setminus C^t} \pi_j \geq z(\pi^t) - \sum_{j \in W \cap C^t} \pi_j^t \Rightarrow \pi^{t+1}$

  Process is re-started until an unblocked core outcome is reached

+
The auction and pricing rules

- Bidder-Pareto-optimal (BPO) core mechanism
  - Seller's final income is equal to the sum of the payment of all winning bidders.

$$\theta^{BPO} = \sum_{j \in W} \pi^{BPO}{}_j$$

  - Advantages: provide an efficient allocation of the lots along with fair payments (core outcome in any combinatorial auction).
  - Drawbacks: complex for bidders to understand and implement.

+ Pricing rules: BPO

Table of bids

	$b_1(S)$	$b_2(S)$	$b_3(S)$	$b_4(S)$
A	20	10		10
B		20	10	10
C	10		20	10
AB				28

Coalitional contribution t=1

	$q_1(S)$	$q_2(S)$	$q_3(S)$	$q_4(S)$
A	10	0		10
B		10	0	10
C	0		10	10
AB				28

$z(\pi^1) = 38 > 30 = \theta^{VCG}$

$\pi_1^2 + \pi_2^2 \geq 38 - 10$

$\pi_1^2 = 14$
$\pi_2^2 = 14$
$\pi_3^2 = 10$

Coalitional contribution t=2

	$q_1(S)$	$q_2(S)$	$q_3(S)$	$q_4(S)$
A	14	0		10
B		14	0	10
C	0		10	10
AB				28

$\theta^{1st} = 60$

$\theta^{BPO} = 38$

$\theta^{VCG} = 30$

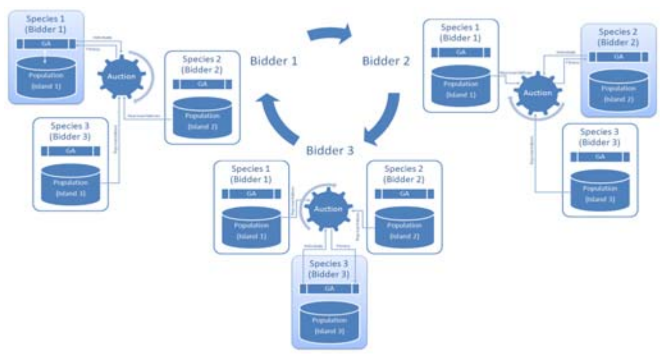
+ Scenarios

- Sale of a portion of the digital dividend that could be representative of any European country of medium to large size.
- Digital dividend, 80 MH:1 lot of 24 MHz (lot A), 2 lots of 16 MHz (lots B, B), and three lots of 8 MHz (lots C, C, C), 23 combinations.
- 1st scenario: Symmetric Bidders
  - 3 weak bidders (leading TV broadcasters)
- 2nd scenario: Asymmetric bidders
  - 2 average bidders (leading TV broadcasters) and 2 strong bidders (incumbent mobile operators)
- 3rd scenario: Strong asymmetries
  - 2 weak bidders (alternative mobile operators) and 2 strong bidders (incumbent mobile operators)
- The spectrum valuation made by each type of participant is determined based on the estimates provided by the British regulator (Ofcom)

Agent-based model

- Modeling the behavior of agents in auctions is not a simple task where the benefit to one participant depends on the behavior of others.
- Agent-based model simulated through co-evolutionary techniques can be more appropriate than evolutionary, Hillis (1990).
- In this model, all agents want to maximize their profits, each of them implements a genetic algorithm (GA) that tries to maximize his fitness value.
- Strategies with higher benefits are more likely to survive and generate new variants of themselves. Strategies that do not derive profits tend to disappear over the generations.

Agent-based model



Example of the agent-based coevolving model (for 3 bidders)

- The different species are allowed to co-evolve over a sufficient amount of time to enable the analysis of the behavior of different participants and auction mechanisms.
- The agents compete against the representatives from the other species.

Computational Experiments

Results in terms of the allocation of lots

Percentage of spectrum allocated to each participant

		B1	Dif*	B2	Dif*	B3	Dif*	B4	Dif*
Symmetric Scenario	COEV_1st	26.47	3.24%**	27.34	5.27%**	26.12	-0.61%	-	-
	COEV_VCG	25.76	0.50%	25.78	-0.74%	26.72	1.67%	-	-
	COEV_BPO	25.48	-0.59%	26.41	1.69%	26.01	-1.00%	-	-
	Efficient	25.64		25.97	5.27%	26.28	-0.61%	-	-
Asymmetric Scenario	COEV_1st	20.75	2.90%	21.13	6.58%**	18.80	-4.51%**	19.31	-4.02%**
	COEV_VCG	20.20	0.16%	20.17	1.73%	19.71	0.12%	19.82	-1.47%
	COEV_BPO	20.53	1.82%	20.25	2.14%	19.40	-1.46%	19.64	-2.39%
	Efficient	20.16		19.83		19.68		19.31	
Strong asymmetries Scenario	COEV_1st	18.77	-34.20%**	18.78	-34.06%**	21.07	83.55%**	21.36	88.90%**
	COEV_VCG	27.00	-5.38%**	26.75	-6.09%**	12.76	11.22%**	13.45	18.95%**
	COEV_BPO	26.44	-7.32%**	26.42	-7.25%**	14.21	23.83%**	12.80	13.15%**
	Efficient	28.53		28.48		11.48		11.31	

Computational Experiments

Results in terms of final payments

Average income of the seller in M Euros sorted from lowest to highest (SD in parentheses)

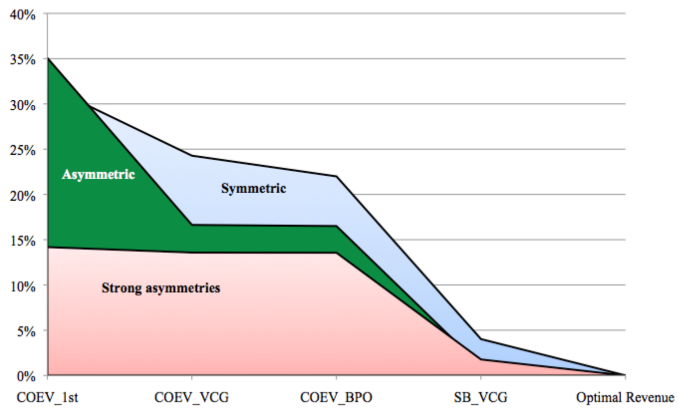
	θ_{COEV}^{1st}	θ_{COEV}^{VCG}	θ_{COEV}^{BPO}	θ_{SB}^{VCG}	**Optimal revenue** θ_{SB}^{BPO}	**Optimal value** θ_{SB}^{1st}
Symmetric scenario	59.48	66.20	68.20	83.92	87.43	188.49
	(15.46)	(15.70)	(15.67)	(17.61)	(15.78)	(9.46)
Asymmetric scenario	101.85	130.76	130.96	155.26	156.85	200.71
	(16.20)	(16.94)	(18.20)	(17.26)	(16.03)	(8.57)
Strong asymmetries scenario	109.36	110.13	110.16	125.18	127.44	173.73
	(11.78)	(8.86)	(9.72)	(7.14)	(8.20)	(5.93)

$$\theta_{COEV}^{1st} \geq \theta_{COEV}^{VCG} \geq \theta_{COEV}^{BPO} \geq \theta_{SB}^{VCG} \geq \theta_{SB}^{BPO} (\text{optimal revenue}) \geq \theta_{SB}^{1st} (\text{optimal value})$$

Computational Experiments

Results in terms of final payments

Percentage of income that the seller loses compared to the optimal revenue.



Concluding remarks

- Combinatorial auctions have become an essential tool to award multiple related lots.
- The rules that the auctioneer establishes are particularly pertinent, as the final results will depend on them.
- This paper has analyzed the effects of the choice of a particular pricing mechanism on the final result in a combinatorial sealed-bid auction.

Concluding remarks

- First-price mechanism, participants try to maximize their expected profit by making the highest bid of those submitted but at the lowest level possible (bidding below their valuations). The final allocation is significantly different from the efficient one and it yields to lower revenues to the seller.
- VCG and the BPO mechanism, the final price paid by the winners depends on the bids made by their rivals. It favor bidding proximate to the private values, thus generating awards that are closer to the efficient outcome.

Concluding remarks

- Comparing VCG and BPO mechanisms:
  - Participants follow the strategic behavior determined by the co-evolutionary system, the differences found between the two systems are only significant in the first scenario.
  - Including only cases where the BPO mechanism finds a blocking coalition: the differences will be statistically significant in all scenarios
- The BPO mechanism ensures fair payments and generates greater revenues for the seller
- However, despite the use of the BPO mechanism, if participants bid in a strategic manner, the seller's income will be reduced significantly with respect to optimal revenue.
- Posible solution: to use a combinatorial auction of multiple rounds such as the clock-proxy auction proposed by Ausubel et al. (2006).

Econ@Tel
A Telecommunications Economic COST Network
WG4 meeting

15 February 2011

Exploring pricing rules in spectrum auctions

José Luis Gómez-Barroso
UNED – Universidad Nacional de Educación a Distancia