

On the ranking strategy in adword auctions

Patrick Maillé and Bruno Tuffin

Telecom Bretagne and INRIA Rennes - Centre Bretagne Atlantique

EconTel WG4 meeting
Jyvaskyla, Finland
February 2011



Outline

- 1 Introduction to adword auctions
- 2 Goal: comparing revenue-based and bid-based schemes
- 3 Model
- 4 Average revenues
- 5 Numerical illustrations and interpretations
- 6 Conclusions/Future activities

Outline

- 1 Introduction to adword auctions
- 2 Goal: comparing revenue-based and bid-based schemes
- 3 Model
- 4 Average revenues
- 5 Numerical illustrations and interpretations
- 6 Conclusions/Future activities

Introduction to adword auctions

- Search engines play a crucial role in the Internet.
- Revenue through advertising slots, usually displayed at the top or right of the search page.
- Advertisers submit bids for relevant keywords only.
- Allocation of slots thanks to adword auctions.
 - ▶ combined revenue of Yahoo! and Google in 2005: \$11 billion in 2005
 - ▶ expected to count for 40% of total advertising revenue.

The screenshot shows a Google search for "used cars usa". The search bar contains the text "used cars usa" and a "Rechercher" button. Below the search bar, it indicates "Rechercher dans : Web Pages francophones Pages : France". The search results are displayed in a list format. The first result is "Used Cars" from MoritzChevrolet.com, with the subtext "Low Prices On Used Cars - Get A Free Internet Quote!". Other results include "New & Used Cars for Sale, Auto Dealers, Car Reviews and Car ...", "Used Cars for Sale, New Cars, & Auto Buying Guide at AutoMailUsa.net", "Used Cars - Cars for Sale - Toyota - Nissan", "New Cars, Used Cars - Find Cars at AutoTrader.com", and "en stock aux Etats-Unis et au Canada - DENKER US CARS | Import New ...". On the right side of the page, there are several sponsored links, including "Used Car in Usa" (100% Free - Cars Classified Ad), "4X4 & Cars for export" (Tax free cars since 1973), "Used Car in Usa" (Trouvez Used car in usa), and "Salvage Auto Auctions USA" (Open 4 Public join to Bid and Buy immediately).

Auction principle (single keyword, K slots)

- Advertisers submit bids for specific keywords.
- Each time there is a search on that keyword:
 - ▶ advertisers are ranked and allocated slots according to a prespecified criterion:
 - ★ bid value (initially for Yahoo!)
 - ★ the revenue they will generate (more or less Google), taking into account the (learned) click-through rate (CTR).
 - ▶ Possible payment rules:
 - ★ *Pay-Per-Impression* (PPI): advertisers charged every time their ad is displayed
 - ★ *Pay-Per-Click* (PPC): advertisers is charged only when the ad is clicked
 - ★ *Pay-Per-Transaction* (PPT): advertisers charged when the click results in a real sell.
 - ▶ Amount to be paid each time?
 - ★ First Price: advertisers pay their bid
 - ★ Generalized Second Price (GSP): they pay the bid of advertiser below them in the ranking
 - ★ Vickrey-Clarke-Groves (VCG) auctions: you pay the opportunity cost that your presence introduce to all other advertisers.
- In use: PPC and GSP.

Outline

- 1 Introduction to adword auctions
- 2 Goal: comparing revenue-based and bid-based schemes**
- 3 Model
- 4 Average revenues
- 5 Numerical illustrations and interpretations
- 6 Conclusions/Future activities

Goal: comparing revenue-based and bid-based schemes

- Most search engines have chosen (or switched to) a revenue-based ranking and charging;
- Is it always relevant?
- What we are going to show:
 - ▶ it depends on the parameters
 - ▶ but bid-based *can* be better than revenue-based.
 - ▶ Therefore: this has to be studied before making a choice or a move.
 - ▶ Two engines may have different optimal strategies when the CTRs of advertisers differ from an engine to another.

Outline

- 1 Introduction to adword auctions
- 2 Goal: comparing revenue-based and bid-based schemes
- 3 Model**
- 4 Average revenues
- 5 Numerical illustrations and interpretations
- 6 Conclusions/Future activities

Basic model

- n advertisers submit bids to search engines for a given keyword.
- Only one slot (then GSP is VCG and truthful-bidding)
- The search engine apply Pay-Per-Click and GSP
- For advertiser i
 - ▶ CTR q_i taken from two classes of advertisers:
 - ★ high-quality advertisers, with CTR q_h
 - ★ low-quality advertisers, with CTR q_l , such that $q_l < q_h$.
 - ▶ The probability α of being a high quality one.
 - ▶ valuation/bid v_i random, distributed according to cdf F and density f , independent of the quality.
- For **bid-based** Pay-Per-Click GSP:
 - ▶ winner: highest bidder $i_b := \arg \max_i v_i$,
 - ▶ charged the second-highest bid $\max_{i \neq i_b} v_i$ at each click
 - ▶ revenue $q_{i_b} \max_{i \neq i_b} v_i$.
- For **revenue-based** Pay-Per-Click GSP:
 - ▶ winner: $i_r := \arg \max_i q_i v_i$,
 - ▶ charged per click: lowest p_{i_r} he could have bid to win
$$p_{i_r} = \frac{1}{q_{i_r}} \max_{i \neq i_r} q_i v_i$$
 - ▶ revenue $q_{i_r} p_{i_r} = \max_{i \neq i_r} q_i v_i$.

Illustration: revenue-based pricing does not always yield a larger revenue

- $n = 2$ advertisers
- advertiser 1 has valuation $v_1 = 0.7$ and CTR $q_h = 0.6$
- advertiser 2 has valuation $v_2 = 0.5$ and CTR $q_l = 0.1$.
- With **bid-based** ranking rule
 - ▶ winner: advertiser 1
 - ▶ revenue will be $v_2 q_h = 0.3$.
- With **revenue-based** ranking rule
 - ▶ advertiser 1 still the winner ($v_1 q_h > v_2 q_l$)
 - ▶ price per click is $v_2 q_l / q_h$
 - ▶ revenue $v_2 q_l = 0.05$.
- Bid-based better.
- Intuition: second-ranked bidder may be a low-quality advertiser, resulting in smaller price per click when using the revenue-based scheme.

Outline

- 1 Introduction to adword auctions
- 2 Goal: comparing revenue-based and bid-based schemes
- 3 Model
- 4 Average revenues**
- 5 Numerical illustrations and interpretations
- 6 Conclusions/Future activities

Average revenues

Proposition

The average revenue under bid-based ranking and charging is

$$R_b = n(n-1)(\alpha q_h + (1-\alpha)q_l) \int x(F(x))^{n-2}(1-F(x))f(x)dx.$$

Proposition

Define

$$\begin{aligned} G(x) &= \alpha F(x/q_h) + (1-\alpha)F(x/q_l) \\ g(x) &= \frac{\alpha}{q_h} f(x/q_h) + \frac{1-\alpha}{q_l} f(x/q_l). \end{aligned}$$

The average revenue under revenue-based ranking and charging is

$$R_r = n(n-1) \int x(G(x))^{n-2}(1-G(x))g(x)dx.$$

Outline

- 1 Introduction to adword auctions
- 2 Goal: comparing revenue-based and bid-based schemes
- 3 Model
- 4 Average revenues
- 5 Numerical illustrations and interpretations**
- 6 Conclusions/Future activities

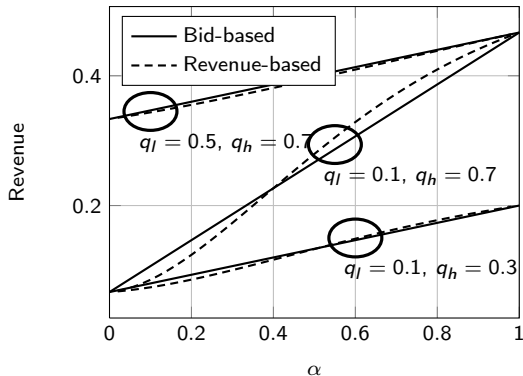
Example description

- $F(x) = x$ for $x \in [0, 1]$
- Then

$$R_b = \frac{n-1}{n+1}(\alpha q_h + (1-\alpha)q_l)$$
$$R_r = n(n-1) \int_0^{q_l} x(\beta x)^{n-2}(1-\beta x)\beta dx$$
$$+ n(n-1) \int_{q_l}^{q_h} x(\gamma x + 1 - \alpha)^{n-2}(\alpha - \gamma x)\gamma dx$$

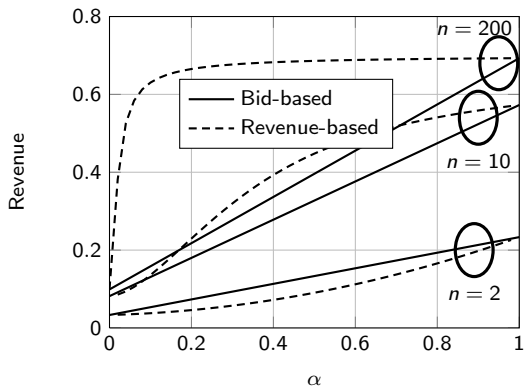
where the last integral can be computed using an integral by part.

Revenue in terms of the proportion α of high-quality users



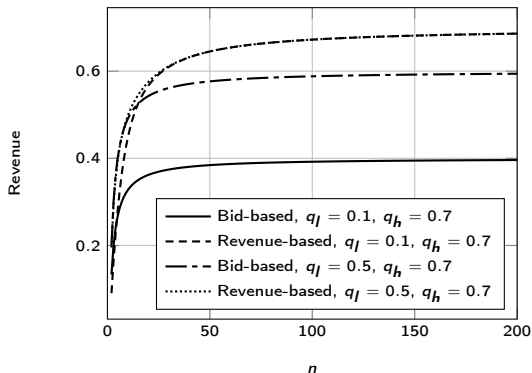
- For low values of α , and low values of q_l , bid-based ranking may produce a larger revenue
- Revenues (obviously) increase with the proportion of high-quality advertisers, and with the CTRs.

Revenues in terms of α for different values of n when $q_l = 0.1$, $q_h = 0.7$



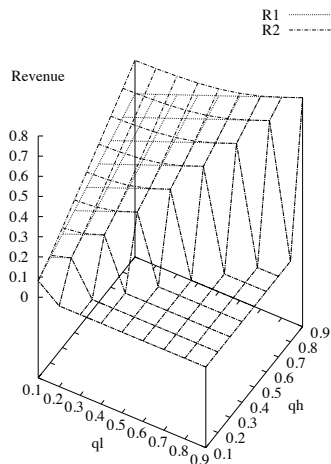
- For a small number of advertisers the chances to get a larger revenue with bid-based ranking is larger (ex: $n = 2$).
- Less likely as the number of advertisers increases (never for $n = 200$).
- Reason: more chances to have a high-quality advertiser in second position with the revenue-based rule.

Revenues in terms of n for different values of q_l , q_h , when $\alpha = 0.5$



- As n increases, revenue-based rule yields a larger revenue.
- Average revenue asymptotically depends on the high-quality advertisers parameters with revenue-based (the two curves tend to coincide) because the second ranked advertisers tend to be a high-quality one.
- Not true for bid-based ranking rule because the valuation is independent of the quality parameter.

Revenues in terms of q_l , q_h when $n = 10$ and $\alpha = 0.5$



- Only considering $q_h > q_l$ (revenues set to 0 otherwise)
- bid-based and revenue-based get closer as q_h and q_l get closer

Outline

- 1 Introduction to adword auctions
- 2 Goal: comparing revenue-based and bid-based schemes
- 3 Model
- 4 Average revenues
- 5 Numerical illustrations and interpretations
- 6 Conclusions/Future activities

Conclusions and our next activities on adword auctions

What we have done:

- determined average revenues when random valuations both for bid-based and revenue-based rankings;
- shown that bid-based ranking may produced a larger revenue than revenue-based.

More generally, we aim at working on competition among search engines:

- Ex: model Google against Yahoo!
- Depending on market shares and interfaces, what is the best ranking strategy?
- Investigate in general games on allocation and pricing *strategies*.