Analysis of Price Competition in a Slotted Resource Allocation Game

Results to be presented at next IEEE INFOCOM Conference

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Seville, April 2008, COST Meeting

"Related" European Initiative: Euro-NF NoE

- `http://euronf.org/`
- NF for *Network of the Future*
- Follows previous NoEs Euro-NGI (*Next Generation Internet*) and Euro-FGI (*Future Generation Internet*)
- WP3: Socio-Economics aspects, divided into:
  - JRA 3.1: Regulation, IPR, Network Neutrality and Governance Policies
  - JRA 3.2: SLAs, Pricing, Quality of Experience
  - JRA 3.3: Cost Models
  - JRA 3.4: Trust, Privacy and Security
- 35 institutions.
- Kick-off meeting last week in Paris.

Outline

Our main interests

- Previous (still active) work: design of Pricing schemes in telecommunication networks to
  - cope with congestion
  - control demand
  - satisfy heterogeneous applications with different QoS
  - yield incentives for a fair share of scarce resources among selfish users.
- Standard tool: non-cooperative game theory.
- New interest: sutdy those pricing schemes in the case of an oligopoly:
  Telecommunication networks, and more specifically the Internet, have switched from an academic to a commercial and competitive network.
  It is primordial to deal with that competition in pricing models when studying price determination:
  - competition may highly affect the results of price determination
  - competition actually introduces an additional level of game, between providers.

Illustrations of competition

- Wired access: DSL users can choose among several competing providers to connect to the Internet.
- Wireless access: a user wishing to connect to a WiFi hotspot may be located in a zone covered by several wireless access providers, and can choose which provider to use for the time of his connection.
- Competition between technologies: possibility to choose between different and competitive transmission platforms: WiFi, WiMAX, 3G, Wired operators.
- There is also a possibility of using several providers at the same time (the so-called multihoming).

Model description

- Time is discretized, divided into slots.
- Set $\mathcal{I} = \{1, \ldots, I\}$ of providers in competition at an access point ($I \geq 2$).
- For each provider $1 \leq i \leq I$:
  - Own capacity C_i per slot,
  - per-packet (or per-unit) price p_i (received or not).
- d_i total demand at provider i,
- so each packet served with probability $\min(C_i/d_i, 1)$.
- Total income of provider i is $d_i p_i$ and the total service rate is $d_i \min(C_i/d_i, 1)$,
- so (per traffic unit) perceived price $\underline{p} = p_i / \min(C_i/d_i, 1) = p_i \max(d_i/C_i, 1)$.
- Remark: separate capacities do not model competition at a WiFi hotspot if providers share the same bandwidth, but when providers are being operated on different frequency channels and using different PHY modes.

Demand and valuation modeling

- Total user demand $D(p)$ continuous and strictly decreasing, with support $[0, p_{\max})$ ($p_{\max} = +\infty$ possible).
- $D(0) > \sum_{i\in\mathcal{I}} C_i$: the total resource available is not sufficient to satisfy the maximum demand level.
- Demand can be interpreted as:
  - heterogeneity in user willingness-to-pay for the service, distributed according to a given distribution
  - decreasingness of D can also stem from the decreasingness of individual demand functions.
- *Marginal valuation* function:

$$v : q \mapsto \inf\{p : D(p) \le q\} = \begin{cases} D^{-1}(q) & \text{if } q \in (0, D(0)) \\ p_{\max} & \text{if } q = 0 \\ 0 & \text{if } q \ge D(0). \end{cases}$$

- $v(q)$ maximum unit price that can be charged for the q^{th} unit of demand (but still selling it) without making the demand decrease.

Valuation and benefit

- $V(q)$ as the sum of the marginal valuations of the q units of users with largest willingness-to-pay:

$$V(q) := \int_{x=0}^{q} v(x)dx.$$

- Other interpretation: total price that the q units of demand with highest marginal valuation are willing to pay to be served.
- For a fixed unit price p, the user surplus equals $V(D(p)) - pD(p)$ if all packets are served.
- Knowing how users will behave, and then $\mathbf{d} := (d_1, \ldots, d_I)$, provider r wants to maximize his net benefit $R_i(p_1, \ldots, p_I) = p_i d_i - \ell_i(d_i)$
  - $p_i d_i$ is the money earned directly from demand
  - $\ell_i(d_i)$ cost for managing a demand level d_i. ℓ_i is nondecreasing, differentiable and convex.

Assumptions

- *Assumption A*. The marginal cost of every provider when his demand equals his capacity is lower than the global marginal valuation of the sum of all provider capacities:

$$\forall i \in \mathcal{I}, \quad \ell_i'(C_i) \leq v\left(\sum_i C_i\right),$$

or equivalently

$$\forall i \in \mathcal{I}, \quad D(\ell_i'(C_i)) \geq \sum_i C_i.$$

- *Assumption B*. For each provider $i \in \mathcal{I}$:

$$\ell_i'(C_i) \leq \left(1 - \frac{C_i}{\sum_{j \neq i} C_j}\right) v\left(\sum_i C_i\right).$$

Goals and questions to be solved

- How, for fixed prices p_i $(i \in \mathcal{I})$, is total demand split among providers?
- Is there a *Nash equilibrium* on prices between providers trying to maximize their net benefit? Is it unique?
  - A Nash equilibrium is a price vector $\mathbf{p} := (p_1, \ldots, p_I)$ such that no provider can increase his own benefit by unilaterally changing his access price, i.e., $\forall i \in \mathcal{I}$, $\forall p \geq 0$,

$$R_i(p_1, \ldots, p_I) \geq R_i(p_1, \ldots, p_{i-1}, p, p_{i+1}, \ldots, p_I).$$

- How far are we from the social optimum obtained in the cooperative case?
- Is there an interest for provider $i \in \mathcal{I}$ to voluntarily declare a false value less than C_i?

Related work

- Many references on pricing in general, much less on competition.
- Most notable references:
  - (Liu et al, INRIA report 2002) studies competition for e-services, with also a kind of Wardrop equilibrium, but where QoS does not depend on demand.
  - (Acemoglu et Ozdaglar, *Path. Oper. Res. 2006*): similar study, but externality is delay on links.
  - Many works on independent and selfish providers on a path, that forward traffic of competitors to ensure end-to-end delivery, not a direct competition for users.
- Our model related to (P. Marbach, *IEEE/ACM Trans. Net.*, vol.12, 2004) where a similar slotted capacity model is used, but with several priority traffic classes, and in the case of a monopoly instead of an oligopoly here.
  - Different goal: study price war between providers instead of price discrimination among users.

Wardrop equilibrium for users

- How is demand split among providers for fixed prices?
- Users infinitely small, their choices do not individually affect the demand levels (and the perceived costs) of the different providers.
- Outcome described by *Wardrop's principle*: demand distributed such that users choose one of the cheapest providers.
- Therefore:
  - user perceived price is the same for all providers having a positive demand:
    $d_i > 0 \Rightarrow p_i \max(1, d_i/C_i) \leq p_j \max(1, d_j/C_j)$
  - and is lower than the unit price p_i of providers i with no demand.
- Total demand level must also correspond to the common perceived price on all providers that receive some demand: $\sum_j d_j = D(\underline{p})$, that is

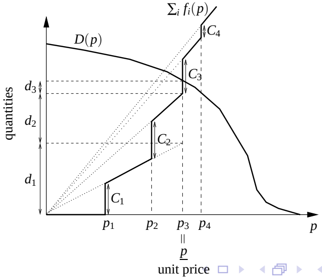
  $d_i > 0 \Rightarrow p_i \max(1, d_i/C_i) = v\left(\sum_j d_j\right).$

Result on Wardrop equilibrium

- *Proposition:* For any capacity and price configuration, there exist a (possibly not unique) user equilibrium demand configuration.
- At a user equilibrium $\mathbf{d}$, the common perceived unit price $\underline{p}$ of providers i with $d_i > 0$ is unique and equals

$$\underline{p} = \inf\{p : D(p) \leq \sum_{i\in\mathcal{I}} f_i(p)\},$$

with $f_i(p) := \mathbb{1}_{\{p \geq p_i\}} \frac{C_i}{p_i} p$.
- Illustration (dynamics):



On the non-uniqueness of Wardrop equilibrium

- Uniqueness of perceived price $\underline{p}$ and total demand $D(\underline{p})$.
- For all providers with price $p_i \neq \underline{p}$, demand d_i is $d_i = f_i(\underline{p})$.
- All providers with $p_i = \underline{p}$ (if any) share the remaining demand $D(\underline{p}) - \sum_{i:p_i<\underline{p}} d_i$, any possible sharing providing a Wardrop equilibrium.
- Non-uniqueness of individual revenue, but total revenue is always the same.

Price competition among providers

- Providers are aware of their advantage of playing first: they take into account users' reaction when determining their price.
- Notation: we use superscript "n" to refer to the values (price, demand, benefit) corresponding to a new situation in contrast to the reference situation.
- *Preliminary property*: given $\mathbf{p} = (p_1, \ldots, p_I)$, if provider $i \in \mathcal{I}$ raises his price to $p_i^n > p_i$ while all other providers $j \neq i$ keep their price to p_j, then
  - the common perceived price (for providers with positive demand) increases: $\underline{p}^n \geq \underline{p}$,
  - if $d_i > 0$ then the demand of provider i strictly decreases: $d_i^n < d_i$.

Price competition, main result

- *Proposition:* Assume $\forall i \in \mathcal{I}$, ℓ_i strictly increasing and convex. Under Assumption B, there exists a *unique Nash equilibrium* on price war among providers, given by

$$\forall i \in \mathcal{I}, \quad \begin{cases} d_i = C_i \\ p_i = p^* \end{cases},$$

where $p^* = v\left(\sum_{j\in\mathcal{I}} C_j\right)$, that is

$$\sum_{i\in\mathcal{I}} C_i = D(p^*).$$

- Proof technical. Sketch on next slide.
- *Remark:* at Nash equilibrium, the corresponding user Wardrop equilibrium is unique.

On the proof

- More details available on my web page:
  `http://www.irisa.fr/armor/lesmembres/Tuffin/Tuffin.htm`
- Paper at:
  `http://www.irisa.fr/armor/lesmembres/Tuffin/Publis/PI1855.pdf`
- First step: $d_i = C_i$ and $p_i = p^* \ \forall i$, with $p^* = v\left(\sum_{i \in \mathcal{I}} C_i\right)$ defines a Nash equilibrium;
- Second step: no other point can be a Nash equilibrium.

Sketch for the first part

- Note that $p_i = p^*$ for all i and $d_i = C_i$
- Let provider 1 go from p^* to $p_1^n \neq p^*$,
  - If $p_1^n < p^*$, then $\underline{p}^n \leq p^*$, $d_1^n = C_1 \underline{p}^n / p_1^n > C_1$, and

    $$R_1^n - R_1 = p_1^n d_1^n - p^* C_1 + \ell_1(C_1) - \ell_1(d_1^n) < 0$$

    because both terms are negative.
  - If $p_1^n > p^*$, then $\underline{p}^n \geq p^*$ and $d_1^n < C_1$. We just study the case $0 < d_1^n$ (no profit otherwise).
    Then $\underline{p} = p_1^n > p^*$ and $\forall i \neq 1, d_i^n = C_i p_1^n / p^*$.
    Using Assumption B, $R_1^n - R_1 < 0$.

Sketch for the second part (uniqueness)

- $\mathcal{I}$ decomposed into subsets: $\mathcal{I} = \mathcal{I}_s \cup \mathcal{I}_0 \cup \mathcal{I}_u$, where

$$\begin{aligned}\mathcal{I}_s &:= \{i \in \mathcal{I} : d_i > C_i\},\\ \mathcal{I}_0 &:= \{i \in \mathcal{I} : d_i = C_i\},\\ \mathcal{I}_u &:= \{i \in \mathcal{I} : d_i < C_i\}.\end{aligned}$$

- First, $\mathcal{I}_s = \emptyset$. If not, if provider $i_s \in \mathcal{I}_s$ increases (alone) its unit price p_{i_s} to $p^n_{i_s} = p_{i_s} + \varepsilon$, his revenue increases too.
- Then, if $\mathcal{I}_u \neq \emptyset$, using the fact that $\mathcal{I}_s = \emptyset$, $i_u \in \mathcal{I}_u$ has interest in decreasing (alone) his price: his revenue would increase (due to increasing demand).

Socially optimal situation

- Social welfare: sum of the utilities of all agents in the system (users+providers).
- Loss of efficiency due to the divergence of user interests: *Price of Anarchy*.
- *Definition*: For a demand configuration $\mathbf{d} := (d_1, \ldots, d_I)$, we call *social welfare* the quantity

$$\mathrm{SW} := V\left(\sum_{i \in \mathcal{I}} \min(d_i, C_i)\right) - \sum_{i \in \mathcal{I}} \ell_i(d_i).$$

- *Proposition*: Under Assumption A, the maximum value of social welfare is reached when $d_i = C_i$ for each provider i.
- *Consequence*: The Nash equilibrium corresponds to the socially optimal situation. Price of Anarchy is 1!

Game on declared capacities

- A provider $i \in \mathcal{I}$ may voluntarily declare a false value $C_i^n \leq C_i$ of his capacity C_i.
- Provider i can easily degrade artificially his service rate, he cannot increase it above his real capacity C_i.
- Opposite effects of lowering one's capacity:
  - the unit selling price at equilibrium increases and the managing cost decreases because the quantity sold decreases
  - whereas on the other hand less quantity sold means less revenue.
- Which effect is more important depends on the *elasticity* of demand, the extent to which total demand is affected by variations of unit price (negative value):

$$\frac{pD'(p)}{D(p)}.$$

Result on declared capacities

- *Proposition*: Under Assumption B, if the absolute value of demand elasticity is larger than 1 for $p \geq p^*$, i.e.

$$\forall p \geq p^*, \quad \frac{-pD'(p)}{D(p)} \geq 1,$$

then no provider can increase its revenue by artificially lowering its capacity.

Conclusions and perspectives

- We have analyzed a pricing game among providers
  - Characterized how demand is split (following Wardrop's principle)
  - found existence and uniqueness of the (characterized) Nash equilibrium of the pricing game
  - shown that it corresponds to the socially optimum point
  - and discussed the interest for users to voluntarily reduce their capacities.
- Perspectives
  - What happens if providers partially share their capacities?
  - What if providers can also play on capacities along with prices?
  - What about the multiclass case?
  - What about the dynamics of the model? How to drive to the equilibrium?

Other general needs

- Determining the utility function, to know the (financial) valuation for a given QoS level
  - Existing work in our group on "Pseudo-Subjective Quality Assesment".
  - Common interest with P. Reichl's here for instance;
  - Relation with WG2 too.
- Help on our multi-level game(s)?