Options in dynamic management of wireless spectrum

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- Spectrum trading represents a convenient solution for both license owners and prospective spectrum users Valletti'01
- A secondary market for spectrum is a reality in many countries

Selling arrangements

Spectrum can be traded in granular quantities

- Frequency-Space-Time (FST) blocks

Doyle and Forde'07

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Spectrum can be traded on different timescales:

- On the spot
  - Limited margins for planning
  - Need for fast switching mechanisms
- Two-stage with options
  - Room for planning
  - Limited initial investment for buyers and chance to renege

Options-based management

- A provider owns N FST blocks
- The provider sells options on each FST block, where each option assigns the buyer the right to buy the block later (option expiry time)
- The buyer pays a price for the option (option price) and, if exercising the option, will pay a pre-determined price for the block (exercise price)
- If the buyer doesn't exercise the option, nothing more is due by it

The overbooking strategy

Overbooking = selling more than what we have

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The net revenues are

1. $R = M \cdot V + B \cdot P$

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The net revenues are

1. $R = M \cdot V + B \cdot P$
2. $R = M \cdot V + N \cdot P - (B - N)C$

The copula model

- The prospective buyers take their decisions independently, but influenced by common factors
- We use a normal copula model to describe the dependence among the decisions taken by the different buyers

$$Y_i = \mathbb{I}(X_i > b_i) = \begin{cases} 0 & \text{no purchase} \\ 1 & \text{purchase} \end{cases}$$

$$X_i = \alpha_i Z + \sqrt{(1-\alpha_i^2)}W_i \qquad Z, W_i \text{ standard normal i.i.d.}$$

The price of options

The price of the option can be set according to the Black-Scholes formula for European call options
The value of the block is assumed to follow a lognormal random walk

Harmantzis and Tanguturi'07

$$V = S_0 G(d_1) - \gamma S_0 \exp(-rT) G(d_2)$$

$G(\cdot)$ = cumulative standard normal distribution

$$d_1 = \frac{-\ln(\gamma) + (r + \sigma^2/2)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{-\ln(\gamma) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

γ = Expected rate of return on the investment

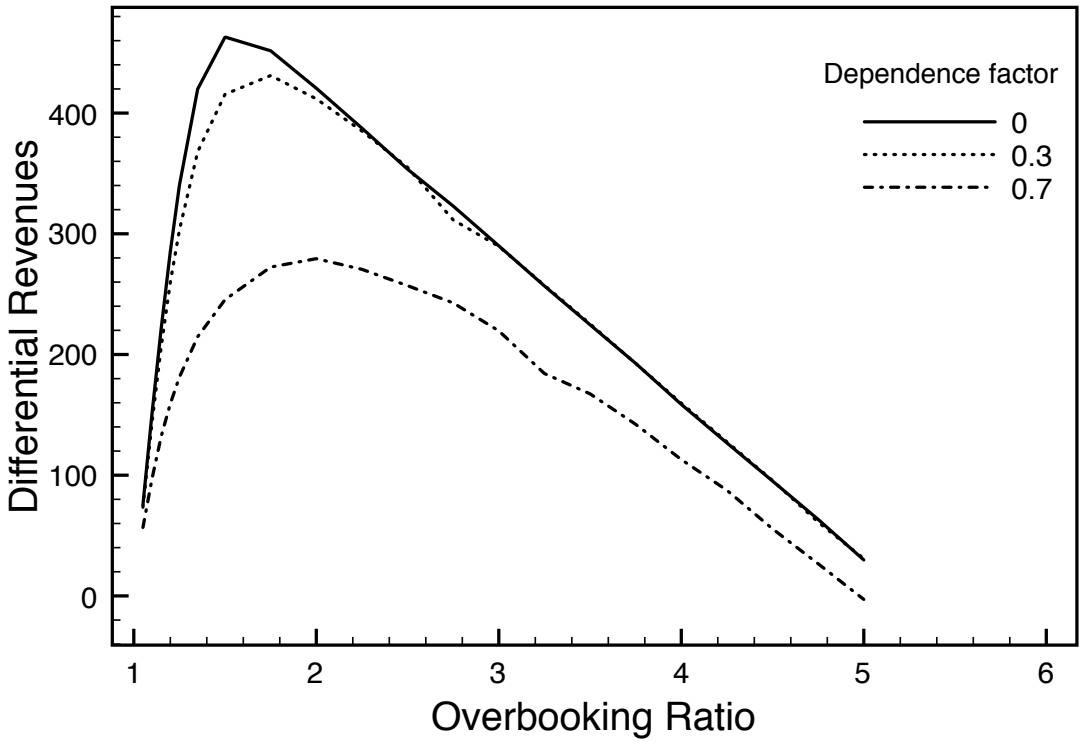
Effectiveness of overbooking

We may use two indicators

1. Probability P_{ob} that the overbooking strategy provides lower revenues than the no-overbooking one
   $P_{ob} < 0.5 \rightarrow$ Overbooking pays
2. Expected differential revenues $\mathbb{E}[R_{ob}]$ (Revenues with overbooking LESS Revenues with no overbooking)
   $\mathbb{E}[R_{ob}] > 0 \rightarrow$ Overbooking pays

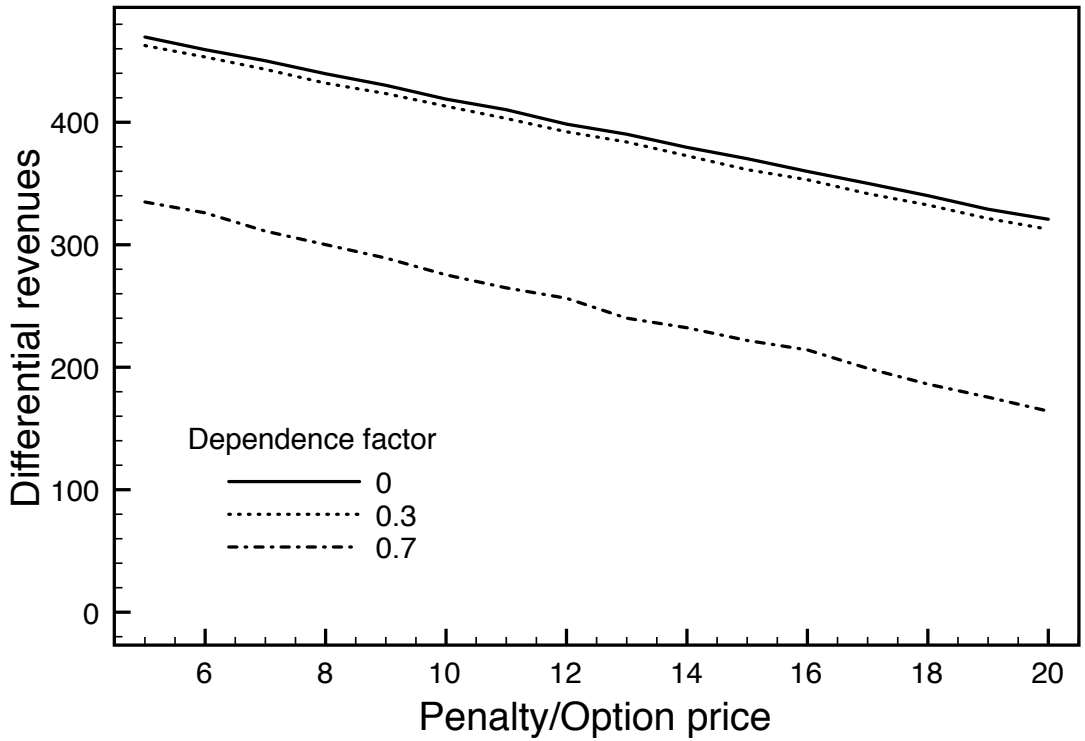
Impact of the overbooking ratio

- Purchase probability $q = 0.75$
- Number of blocks $N = 20$
- Penalty/Option price $C/V = 10$
- Block price/Option price $P/V = 100$



Impact of the penalty

- Purchase probability $q = 0.75$
- Number of blocks $N = 20$
- Overbooking ratio $M/N = 2$
- Block price/Option price $P/V = 100$



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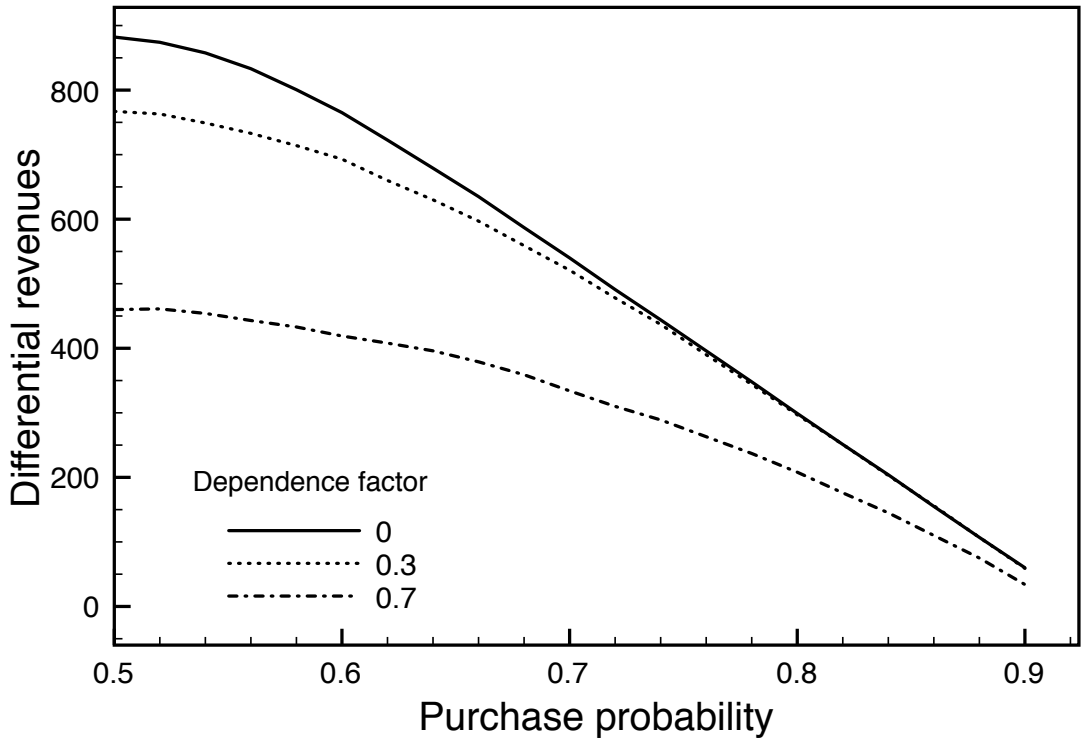

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- When the overbooking ratio is lower than a threshold (in the 2-3 range), the overbooking gain increases significantly, decaying fast as that threshold is exceeded
- The correlation among the different spectrum consumers negatively affects the gain due to the overbooking strategy even for small values of the purchase probability